A Probabilistic Load Flow with Consideration of Network Topology Uncertainties

Liang Min, Member, IEEE, and Pei Zhang, Senior Member, IEEE

Abstract—Our past research proposed to apply Cumulants and Gram-Charlier expansion method to perform probabilistic load flow studies with consideration of generation and load uncertainties. This paper proposed a new method to improve the previous PLF computation method in order to model the network topology uncertainties. This innovative method uses Distribution Factor concept to model the impact of network uncertainties as a linear function of power injections. Maintaining the linear relationship between line flows and power injections enables applying Cumulants and Gram-Charlier expansion method to compute probabilistic distribution functions of transmission line flows. The proposed method is examined using IEEE 30-bus test system. Numerical comparison with Monte Carlo simulation method is also presented in this paper. Study results indicate that the proposed method has significantly reduced the computational efforts while maintaining a high degree of accuracy.


I. INTRODUCTION

Recent changes in the U.S. electric power industry has caused dramatic increases in the use of the transmission system. Transmission planning has become an increasingly important topic due to the fact that federally mandated open access transmission has created a more competitive bulk power market place. This competitive market place brings great challenges to system planning.[1]

Computation of power flows is one of the major tasks in system planning studies. Deterministic load flow study requires specific values for loads, generation inputs and network conditions. In an open access environment, this information is not as certain as it used to be when the power system was a vertically integrated system. In system expansion planning, it is desirable to assess bus voltages and line flows for a range of load and generation conditions. To carry out conventional load flow computations for every possible or probable combination of loads and generation, and network topology is impractical because of the extremely large computational effort required. Performing probabilistic load flow studies gives system planning engineers a better feel of future system conditions and will provide more confidence in making judgments concerning investment.

Application of probabilistic analysis to the power system load flow study was first proposed by Borkowa in 1974.[2] Since then, there are two ways of adopting probabilistic approach to study load flow problems: Stochastic Load Flow (SLF)[3]-[6] and Probabilistic Load Flow (PLF)[7]-[12]. In SLF study, the load and generation at an instant time are treated as random variables. SLF investigates the impact of this uncertainty to the output of conventional power flow at each instant of time. Consequently, SLF deals with short time uncertainties and is useful for system operational planning. Since this paper investigates the effects of load and generation uncertainty over a long-term period on the adequacy of transmission network, we adopt PLF approach for planning study purpose.

Many PLF methods have been proposed to study load flow uncertainty problem. These methods can be classified into two categories: simulation method and analytical method. Monte Carlo (MC) simulation is the widely used simulation method.

In order to reduce the computational effort, the analytical methods were proposed [2, 11-17]. In [2], Borkowska first proposed a DC load flow model to take node data uncertainty into account and to find the distributions of branch flows. Reference [10] used a direct approach based on the principle of statistical least square estimation to compute the effects of uncertainties in input data on all output quantities and to obtain the expected value and variance of the solution of a load flow problem. A discrete frequency domain convolution technique by applying fast Fourier transformations and linearized power flow equation was proposed in [12] to increase the computation accuracy and to find the PDF of all output quantities. Reference [13] combined the concept of Cumulants and Gram-Charlier expansion theory to consider the bus injection uncertainties and to achieve enough accuracy with less computation effort to compute the approximate PDF and cumulative distribution function of network branch flows. A new boundary load flow method based on fuzzy/interval numbers was proposed in [14] to find the accurate boundary load flow solutions. Allan and da Silva [15] proposed a new PLF algorithm based on linearized models to account for the nonlinear network equations and to compute the distributions of output quantities. Wang and Alvarado [16] used interval arithmetic to consider the uncertainty of nodal data and to find all possible solutions included within the bounds given by interval arithmetic. A new PLF method based on a description of bus power injections as random variables was proposed in [17] to consider the bus power injection uncertainties and operating practice of power systems. The main advantage of the analytical methods mentioned above is to avoid the computer simulations, but more assumptions and complex mathematical algorithms are required for these methods.

L. Min and P. Zhang are with Electric Power Research Institute (EPRI) (e-mail: lmin@epri.com, pzhang@epri.com).
This paper is an extension of our previous work [13]. In this paper, the network topology uncertainties caused by branch outages are taken into account. The distribution factor is used to model the impact of branch outages as a linear function of power injections so that cumulants and Gram-Charlier expansion theory can be used when computing cumulative distribution of the line flows. This paper is organized as follows: Section II describes the power flow formulation. Section III proposes the PLF computation method. Section IV presents the calculation procedures of this method. IEEE 30-bus test system is used to demonstrate the proposed method in Section V. Section VI concludes the paper.

II. POWER FLOW FORMULATION

The load flow study computes the steady-state solution of the power system. In a deterministic load flow study, the known quantities are the injected active powers \( P_i \) at all busbars (where \( P \) and \( Q \) or \( P \) and \( V \) are known) except the slack bus, the injected reactive powers \( Q_j \) at all load busbars (where \( P \) and \( Q \) are known) and the voltage magnitude \( V_i \) at all generator busbars (where \( P \) and \( V \) are known).

\[
P_i = g_i(\delta_i, \delta_j, \delta_n, V_i, V_j, V_n) \quad (1)
\]

\[
Q_j = h_j(\delta_i, \delta_j, \delta_n, V_i, V_j, V_n) \quad (2)
\]

where \( i = 1, 2, \ldots, n \).

Since equations (1) and (2) are nonlinear in terms of the voltage magnitudes and angles (considered as state variables in this paper), the numerical solution must be based on an iterative method.

In PLF studies, the input variables \( P_i \) and \( Q_j \) are defined by probabilistic density functions. It is preferable to apply linear approximation to equations (1) and (2) so that the state variables could be solved as a linear combination of input variables. This, in turn, will not only allow us to solve load flow equations through fast direct methods but will also permit application of convolution techniques to arrive at the probabilistic description of the variables of interest.

In long term power system planning, the main problem is to locate transmission and generation facilities in the appropriate places and in time to satisfy the customer’s real power demand. Based on these considerations, this paper adopted dc load flow in the formulation of PLF problems.

In DC power flow, the line flow from bus \( i \) to bus \( j \), \( P_{ij} \), is given by

\[
P_{ij} = \frac{\theta_i - \theta_j}{X_{ij}} \quad (3)
\]

where

\( x_{ij} \) line inductive reactance in per unit;
\( \theta_i \) phase angle at bus \( i \);
\( \theta_j \) phase angle at bus \( j \).

The total power flowing into bus \( i \), \( P_i \), is the algebraic sum of generation and load at the bus and is called a bus power injection. It must equal the sum of the power flowing away from the bus on the transmission lines, so

\[
P_i = \sum_j P_{ij} = \sum_j \frac{\theta_i - \theta_j}{X_{ij}} \quad (4)
\]

This can be expressed as a matrix equation

\[
\begin{bmatrix} P_i \\ \vdots \end{bmatrix} = B \begin{bmatrix} \theta_i \\ \vdots \end{bmatrix} \quad (5)
\]

where the elements of the susceptance matrix \( B \) are functions of the line reactance \( x_{ij} \). The \( B \) matrix is singular, but by declaring one of the buses to have a phase angle of zero and eliminating its row and column from \( B \) the reactance matrix \( X \) can be obtained by inversion.

The resulting equation then gives the bus phase angles as a function of the bus injection.

\[
\begin{bmatrix} \theta_i \\ \vdots \end{bmatrix} = X \begin{bmatrix} P_i \\ \vdots \end{bmatrix} \quad (6)
\]

where the injection at the zero phase angle bus is simply the negative sum of all other bus injections in the system.

III. PROPOSED COMPUTATION METHOD

A. Power Transfer Distribution Factor

The linearity property of the DC power flow model can be used to find the injection amount that would contribute to a specific power flow. Consider a bus \( m \) and a line connecting buses \( i \) and \( j \). Following Wood and Wollenberg’s method [18], the coefficient of the linear relationship between the incremental amount of an injection and the incremental flow on a line is called the (incremental) power transfer distribution factor (PTDF).

\[
PTDF_{ij,m} = \frac{X_{im} - X_{jm}}{x_{ij}} \quad (7)
\]

where \( x_{ij} \) is the reactance of the transmission line connecting bus \( i \) and bus \( j \); \( X_{im} \) and \( X_{jm} \) are the elements on the \( i \)th row and the \( m \)th column of the bus reactance matrix \( X \). For brevity, the sensitivity is called “the PTDF from \( m \) to line \( ij \).”

Therefore, the line flow from bus \( i \) to bus \( j \), \( P_{ij} \), can be expressed as

\[
P_{ij} = \sum_m PTDF_{ij,m} \cdot P_m \quad (8)
\]

where \( P_m \) is the injection at bus \( m \).

B. Line Outage Distribution Factor

When an outage occurs, the power flowing over the outage line is redistributed onto the remaining lines in the system. \( LODF_{jr,s} \) is the coefficient of power flow change over the line from bus \( r \) to \( s \) due to the outage of the line from bus \( r \) to \( s \).

\[
\Delta P_{ij,rs} = LODF_{ij,rs} \cdot P_{rs} \quad (9)
\]

where \( P_{rs} \) the power flowing on the line from bus \( r \) to bus \( s \) before its outage.

The LODF is the measure of redistribution that can be computed using the following equation

\[
LODF_{ij,rs} = \frac{x_{ir}(X_{sr} - X_{rs} - X_{jr} + X_{jr})}{x_{ij}[x_{rs} - (X_{jr} + X_{sr} - 2X_{jr})]} \quad (10)
\]

where \( x_{ij} \) and \( X_{rs} \) are as in (7).
If we use Equation (8) to represent the flow over the line from bus \( r \) to bus \( s \), Equation (9) can be rewritten as

\[
\Delta P_{rs} = L O D F_{rs} \cdot \sum P T D F_{rs} P_m
\]

(11)

The impact of line outages is now converted as a linear function of power injections.

C. Model the Uncertainty of Line Outage

When the uncertainties of line outages are taken into account, the line flow from bus \( i \) to bus \( j \), \( P_{ij} \), is given by

\[
P_{ij} = P_{ij}^{nor} \cdot p_{ij}^{nor} + \sum P_{ij}^{los} \cdot p_{ij}^{los}
\]

(12)

where

- \( P_{ij}^{nor} \) the power flow over the transmission line connecting bus \( i \) and bus \( j \) under normal condition;
- \( p_{ij}^{nor} \) the probability of normal condition;
- \( P_{ij}^{los} \) the power flow over the transmission line between bus \( i \) and bus \( j \) when the outage happens on the line between bus \( r \) and bus \( s \);
- \( p_{ij}^{los} \) the outage probability of the line between bus \( r \) and bus \( s \).

The line flow \( P_{ij} \) with consideration of network topology uncertainties can expressed as

\[
P_{ij} = \sum_{m} PTDF_{ij,m} \cdot P_m \cdot p_{ij}^{nor} + \sum_{v} \left( P_{ij}^{los} + LODF_{ij,m} \cdot P_m \right) \cdot p_{ij}^{los}
\]

\[
= \sum_{m} \left( PTDF_{ij,m} \cdot p_{ij}^{nor} + \sum_{v} \left( PTDF_{ij,m}^{los} \cdot P_m \right) \cdot p_{ij}^{los} \right) \cdot P_m
\]

\[
= \sum_{m} h_{ij,m} \cdot P_m
\]

(13)

Therefore, (12) is an approximation of the line flow with the consideration of only N-1 contingencies. The effects of N-k contingencies are ignored because the high order contingencies are with very low probabilities compared with N-1 contingencies.

IV. COMPUTATION PROCEDURES

The Cumulants and Gram-Charlier expansion theories reviewed in [13] are adopted to approximate a distribution by a standard normal distribution. Based on the above method and Gram-Charlier expansion theory, the procedures of calculating PDF of line flows is summarized as follows, 1) Given the probabilistic description of generation and load, calculate the vth (\( v > 0 \)) moments of injected active power by the following equation:

\[
\alpha_v = E(\xi^v) = \int_{-\infty}^{\infty} x^v dF(x)
\]

where \( E(\cdot) \) is the mathematical expectation operator; \( F(x) \) is a continuous distribution function with a random variable \( \xi \).

2) Compute the cumulants of injected power according to the following relationship between cumulants and moments:

\[
\gamma_1 = \alpha_1 = n
\]

\[
\gamma_2 = \alpha_2 - \alpha_1^2
\]

\[
\gamma_3 = \alpha_3 - 3\alpha_1\alpha_2 + 2\alpha_1^3
\]

\[
\gamma_4 = \alpha_4 - 3\alpha_1\alpha_2^2 + 4\alpha_2\alpha_1^2 + 3\alpha_1^4
\]

……

where \( m \) denotes the mean value; \( \alpha_v \) is the vth (\( v > 0 \)) moment.

3) Compute the cumulants of line flow according to equation (13). For the \( i^{th} \) line flow,

\[
P_{linei} = h_{ij}P_j + h_{ij}P_j + \cdots + h_{ij}P_m.
\]

For the cumulants related with the \( i^{th} \) line flow,

\[
\gamma_{rs} = h_{ij}^i\gamma_{ij}^{(1)} + h_{ij}^i\gamma_{ij}^{(2)} + \cdots + h_{ij}^i\gamma_{ij}^{(n)}
\]

where \( v = 1, 2, \ldots, 9 \).

4) Compute central moments of each line based on the following equation.

\[
b_1 = 0
\]

\[
b_2 = \gamma_2 = \sigma^2
\]

\[
b_3 = \gamma_3 + 3\gamma_2
\]

\[
b_4 = \gamma_4 + 10\gamma_2\gamma_2
\]

\[
b_6 = \gamma_6 + 15\gamma_2\gamma_4 + 10\gamma_2\gamma_2 + 15\gamma_2
\]

…….

where \( \sigma \) denotes standard deviation.

5) Calculate the Gram-Charlier expansion coefficients using the following equation:

\[
c_1 = c_2 = 0
\]

\[
c_3 = \frac{\beta_3}{\sigma^3}
\]

\[
c_4 = \frac{\beta_4}{\sigma^4} - 3
\]

\[
c_5 = \frac{\beta_5}{\sigma^5} + 16\frac{\beta_3}{\sigma^3}
\]

\[
c_6 = \frac{\beta_6}{\sigma^6} - 15\frac{\beta_4}{\sigma^4} + 30
\]

……

6) The cumulative distribution function and probabilistic density function of line flows can be obtained using the following two equations:

\[
F(x) = \Phi(x) + \frac{c_2}{2!} \Phi^{(1)}(x) + \frac{c_2}{3!} \Phi^{(2)}(x) + \cdots
\]

\[
f(x) = \varphi(x) + \frac{c_2}{2!} \varphi^{(1)}(x) + \frac{c_2}{3!} \varphi^{(2)}(x) + \cdots
\]

where \( \Phi(x) \) and \( \varphi(x) \) represent the cumulative distribution function (CDF) and probabilistic density function (PDF) of normal distribution with zero mean and unit variance.

V. CASE STUDY AND RESULT COMPARISON

The method described in the preceding sections is examined using the IEEE 30-bus test system. The network diagram is shown in Figure 1. All loads are assumed to be normally distributed, their mean values are equal to the base case bus loads, and their standard deviations are all equal to 10% of their mean values. To model generation uncertainties, the generation units’ forced outage rate (FORs) are set as 0.1. The capacity can be obtained using the binomial distribution. To model transmission uncertainties, branches FORs are set as 0.2%.
In order to demonstrate accuracy and efficiency of proposed method, it is compared with Monte Carlo simulation. Monte Carlo simulation repeats the process of deterministic load flow computation using, in each simulation, a particular set of values of the random variables generated in accordance with the corresponding probability distributions. With consideration of accuracy comparison, the simulation sets the number of trials at 5000.

Table 1 lists the calculation time using different approaches. All calculations are conducted on a Pentium IV 2.4-GHz personal computer using a program developed in MatLab. It can be seen that, depending on the order of Gram-Charlier expansion, the new method proposed in this paper is much faster than Monte Carlo Simulation with 5000 iterations.

<table>
<thead>
<tr>
<th>Methods</th>
<th>Computation Time (seconds)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monte Carlo (5000 iterations)</td>
<td>156.78</td>
</tr>
<tr>
<td>Cumulants &amp; Gram-Charlier (3rd)</td>
<td>6.55</td>
</tr>
<tr>
<td>Cumulants &amp; Gram-Charlier (6th)</td>
<td>6.60</td>
</tr>
<tr>
<td>Cumulants &amp; Gram-Charlier (9th)</td>
<td>6.73</td>
</tr>
</tbody>
</table>

Figure 2 and Figure 3 shows the cumulative distribution curves of the line flow from bus 1 to bus 2 and line flow from bus 21 to bus 22, respectively. In order to display the graph clearly, we only show 3rd, 6th and 9th orders of combined Cumulants and Gram-Charlier Expansion method. It can be seen that, with comparison of Monte Carlo Simulation, combined Cumulants and Gram-Charlier Expansion method can precisely calculate the CDF of line flows.

Average Root Mean Square (ARMS) error is computed using the Monte Carlo 5000 iteration results as reference. In order to demonstrate the accuracy of this method, the relevant ARMS results of power flows on the line Bus 1 - Bus 2 and line Bus 21 - Bus 22 are shown in Table 2 and Table 3, respectively. ARMS is defined as:

$$ARMS = \sqrt{\frac{\sum_{i=1}^{N} (CG_i - MC_i)^2}{N}}$$  \hspace{1cm} (35)

where $CG_i$ is the $i$th point's value on the cumulative distribution curve calculated using the method of combined Cumulants and Gram-Charlier expansion; $MC_i$ is the $i$th point's value on the cumulative distribution curve calculated using the Monte Carlo method; $N$ represents the number of points.

<table>
<thead>
<tr>
<th>Methods</th>
<th>ARMS</th>
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<tbody>
<tr>
<td>Cumulants &amp; Gram-Charlier (3rd)</td>
<td>0.37%</td>
</tr>
<tr>
<td>Cumulants &amp; Gram-Charlier (6th)</td>
<td>0.39%</td>
</tr>
<tr>
<td>Cumulants &amp; Gram-Charlier (9th)</td>
<td>0.37%</td>
</tr>
</tbody>
</table>
In planning, we are interested in the 10% and 90% confidence levels that the line flow will not exceed because this would indicate roughly the desired capacity of the path. Consequently, accurate estimation of MW value at 10% and 90% confidence levels has important meaning to system planning engineers. For example, the MW value at 10% and 90% confidence levels has important meaning to system security level and the maximum power rating of the line with uncertainties as a linear function of power injections.

This paper proposed a new method to improve the previous PLF computation method in order to consider the network topology uncertainties. This innovative method uses distribution factor concept to model the impact of network topology uncertainties. This innovative method uses cumulative distribution function of transmission line flows, multilinearisations, and applying this method to large interconnected systems.

In conclusion, computation results have proved that our proposed method enables system planners to estimate 10% and 90% confidence levels accurately enough to determine the security level and the maximum power rating of the line with confidence.

VI. CONCLUSIONS

This paper proposed a new method to improve the previous PLF computation method in order to consider the network topology uncertainties. This innovative method uses distribution factor concept to model the impact of network uncertainties as a linear function of power injections. Maintaining the linear relationship between line flows and power injections allows us to apply Cumulants and Gram-Charlier expansion method to computing probabilistic distribution functions of transmission line flows.

Simulation results on the IEEE 30-bus system are presented. The results are compared with Monte Carlo simulation for validation. With the comparison of Monte Carlo simulation results, the new method is able to accurately approximate the cumulative distribution function of transmission line flows, while significantly increase the computation efficiency. Results have shown that the proposed method is much faster than Monte Carlo simulation. Theoretically, the computation burden of the proposed method will not increase dramatically with the increase of system size. Future research will consider applying this method to large interconnected systems.

VII. REFERENCES


VIII. BIOGRAPHIES

Liang Min (M’07) received his Ph.D degree from Texas A&M University, College Station, Texas. He is currently a project manager with the Electric Power Research Institute (EPRI), Palo Alto, California. His current research interest is in interregional operation and planning.

Pei Zhang (SM’05) received his Ph. D. degree from Imperial College of Science, Technology and Medicine, University of London, U.K. He is currently a Sr. Project Manager in Grid Operations and Planning group with the Electric Power Research Institute (EPRI), Palo Alto, California, USA. His research interests include application of probabilistic method to system planning area.