Transmission Network Expansion Planning with a Hybrid Meta-heuristic Method of Parallel Tabu Search and Ordinal Optimization

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Abstract—This paper proposes a hybrid meta-heuristic method of parallel tabu search (PTS) and ordinal optimization (OO) for transmission network expansion planning in power systems. It determines the optimal structure that keeps the balance between generations and loads. The formulation is expressed as a combinatorial optimization problem that is very hard to solve. PTS is one of meta-heuristics that is useful for solving a combinatorial optimization. To speed up computational time of PTS, OO is used to reduce the number of solution candidates in a probabilistic way. The proposed method with OO-TS is successfully applied to a sample system.

Index Terms—Transmission Network Expansion Planning, Meta-heuristics, Combinatorial Optimization, Parallel Tabu Search, Ordinal Optimization

I. INTRODUCTION

This paper proposes a hybrid meta-heuristic efficient method for transmission network expansion planning. The objective of transmission network expansion planning is to construct the optimal network that satisfies generation and loads in the future network. The increase of loads requires the new facilities in the existing network. As a result, it is a key issue to extend the existing network in a way that the cost is minimized while meeting the constraints. However, many solution candidates make the problem more complicated in a sense of combinatorial optimization. In other words, it is one of the most difficult combinatorial optimization problems in power systems. The conventional methods may be classified as follows:

- Enumeration methods
- Heuristics
- Meta-heuristics

The enumeration methods and heuristics have a drawback that the obtained solution is locally optimal or that they are not applicable to real-size systems [1, 2]. On the other hand, meta-heuristics is more attractive in a sense that a better solution is obtained in real-size systems. Meta-heuristics means an optimization method that repeatedly makes use of simple rules or heuristics to evaluate a globally optimal or its approximate solutions [13]. As typical meta-heuristics, the followings are well-known in the engineering fields:

- Simulated Annealing (SA)
- Genetic Algorithm (GA)
- Tabu Search (TS)

SA stems from the analogy of heat bath in the process of metal annealing [9]. It introduces a parameter called temperature to control the search space from high to low temperature. It makes use of probabilistic search to escape from a local minimum. However, it has a drawback that it is very time-consuming to evaluate the solution in large-scaled systems. GA is based on the natural selection of biology in a way that better solutions are preserved for the next generation and evolved to obtain better solutions [10]. It carries out multi-point probabilistic search and employs the genetic operations such as crossover, mutation, reproduction, etc. to diversify the solution candidates. However, it has difficulty in tuning up the parameters of the genetic operators in large-scaled systems. TS is the analogy of working memory in cognitive psychology [11, 12]. It is the extension of the hill-climbing method (HLM) that creates the solution candidates around a solution and moves to the best solution in the neighborhood. The main difference between TS and HLM is that TS has the function of adaptive memory called tabu list to escape from a local minimum. It plays a key role to preserve some attributes for a while. Regarding applications of meta-heuristics to transmission network expansion planning,

Table 1 Comparison of Meta-heuristics

<table>
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<tr>
<td>SA</td>
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<td>Less</td>
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<td>GA</td>
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<td>Population Crossover Mutation</td>
<td>Less</td>
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</tr>
<tr>
<td>TS</td>
<td>Adaptive Memory</td>
<td>Tabu Length</td>
<td>More</td>
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Romero, et al. presented method with SA [3, 5]. Although it is expected that SA gives a better solution, it is more time-consuming and often affected by the initial solutions in realistic problems. A TS-based transmission network expansion planning method was developed to evaluate a better solution [4, 8]. Gil and da Silva applied GA to transmission network expansion planning [8]. Compared with SA and GA, TS gives better results in terms of solution accuracy and computational time. However, the recent results have shown that TS is inclined to be time-consuming in large-scaled systems [14-19]. That is because the creation of the neighborhood solutions needs heavy computational effort. Table 1 shows a comparison of meta-heuristics, where the features of SA, GA and TS are given. The table shows that TS is better than SA and GA.

In this paper, an efficient TS-based method is proposed for transmission network expansion planning. To improve the performance of TS, this paper employs Parallel Tabu Search (PTS) that has a couple of strategies [15]. One is to decompose the neighborhood of TS into subneighborhoods so that computational time is reduced. The other is to use multiple tabu lengths to diversify solution candidates in a way that different tabu lengths diversify the solution candidates. In addition, ordinal optimization (OO) [20] is used to speed up the calculation of the neighborhood solutions through the random selection. It is a complementary technique that improves the performance of search process in discrete optimization problems. In this paper, an OO-based technique is used to speed up the performance of PTS. The proposed method is successfully applied to sample systems.

II. TRANSMISSION NETWORK EXPANSION PLANNING

This section describes the mathematical formulation of transmission network expansion planning. It aims at constructing the optimal network to meet the requirements of loads and generation with the transmission lines. Mathematically, it results in a combinatorial optimization problem. The formulation may be written as follows:

\[
v = \sum_i \sum_j c_{ij} n_{ij} + \sum_i a_i r_i \rightarrow \min
\]

Cost Function:

Constraints:

\[
B(x + y) \theta g = -d
\]

\[
|\theta - \theta_0| \leq \Phi_i
\]

\[
0 \leq g \leq g_{\text{max}}
\]

\[
0 \leq n_{ij} \leq n_{ij,\text{max}}
\]

where

- \( v \): cost function
- \( c_{ij} \): installation cost at branch \( ij \)
- \( a_i \): penalty coefficient
- \( r_i \): amount of overload
- \( B(*) \): susceptance matrix
- \( x \): total susceptance of branch \( ij \)
- \( y^0 \): initial susceptance
- \( \theta \): nodal voltage angle vector
- \( \Phi_i \): maximum power flow at branch \( ij \)
- \( r_i \): susceptance of new lines
- \( g \): generator generation vector
- \( d \): demand generation vector
- \( n_{ij,\text{max}} \): upper bound of \( n_{ij} \)

The first term of (1) shows the installation cost of branches and the second gives the penalty for overload violation. Eqn. (2) denotes the constraint on the DC load flow. Eqns. (3) and (4) show the constraints on the limitation of branches and generations, respectively. Eqn. (5) gives the constraint on the number of installed lines. In this paper, the location of new lines is determined by minimizing the cost function of (1). Thus, the objective of the formulation is that the second term approaches zero in (1). It is hard to solve the formulation directly. So, the penalty function method is used for (1)-(5) after evaluating the state of lines. The process is repeated until the termination conditions are satisfied. To solve the problem, this paper proposes a hybrid method of PTS and OO.

III. PARALLEL TABU SEARCH

This section outlines parallel tabu search (PTS) that improves the performance of TS. PTS is an extension of TS for solving a combinatorial optimization problem. TS is based on the hill-climbing method (HCM) of local search that repeatedly evaluates solutions by creating the neighborhood around a solution. It is well-known that HCM often gets stuck in a local minimum. To reduce the cost function in a sense of global optimization, a new methodology is required in the engineering fields. To evaluate a global optimal solution or its approximate one, meta-heuristics needs a couple of strategies, i.e.

- Intensification
- Diversification

The former means a method that intensively finds out better solutions around a solution. The latter implies a method that evaluates a better solution through the diversity of solution candidate to escape from a local minimum. Regarding TS, the neighborhood search corresponds to the intensification while the function of the tabu list means the diversification in a way that the algorithm moves to other solutions to avoid a local minimum. Specifically, TS has the following features:

- It has only one parameter called tabu length. As a result, it is much easier to tune up it.
- Unlike SA and GA, the algorithm makes use of deterministic local search.
It has adaptive memory called tabu list to escape from a local minimum.

The algorithm is a transition type rather convergence one in a sense that the converged solution does not exist. Namely, the termination conditions are freely set within certain iterations.

However, the recent studies have shown that TS has limitations that better solutions are evaluated in large-scaled problems. In other words, the diversification and the computational efficiency deteriorate in such problems. To overcome the problem, PTS was developed. It has a couple of strategies:

- decomposition of the neighborhood in TS into subneighborhoods
- use of multiple tabu lengths

Let us consider a case where two subneighborhoods are used. As shown in Fig. 1(a), the neighborhood is decomposed into two subneighborhoods, where CPUs 1 and 2 are assigned to subneighborhoods 1 and 2, respectively. If the neighborhood for the Hamming distance of 1 is used, the solution candidates in subneighborhoods 1 and 2 are independently calculated so that computation time is reduced. Fig. 1(b) shows a case where two tabu lengths are given to an initial solution. The different tabu lengths bring about different search direction so that the diversity of solutions candidates is realized and better solutions are obtained. Therefore, it is expected that PTS is more efficient than TS.

IV. PROPOSED METHOD

It this section, ordinal optimization (OO) is briefly outlined. OO developed by Ho, et al. focuses on evaluating enough good feasible solutions in discrete optimization problems rather than the best one [20]. To some extent, it deteriorates the solution accuracy, but speeds up computational time remarkably. In general, the number of combinations exponentially increases with the size of a problem. As a result, more reliable methods are required to evaluate the solution within a certain time. OO evaluates the solution in a large-scaled problem by calculating a subspace that includes enough good solutions in the feasible solution space. That is referred to as goal softening. OO has two-staged of goal softening.

Goal Softening at Stage 1

Now, let us select $N$ solutions in feasible solution subspace $\Theta$. Stage 1 selects subspace $\Theta'$ where at least one solution is included in the top of $\alpha\%$ with probability $p$. Then, $N$ is determined by the following equation:

$$N \geq \frac{\log(1-p)}{\log(1-\frac{\alpha}{100})}$$

Fig. 2 Concept of Ordinal Optimization

Goal Softening at Stage 2

Goal softening at Stage 2 is to determine subset $S$ that has at least $k$ solutions in $G$, where subset $S$ is the subset with $n_s$ elements that are randomly selected in subset $\Theta'$, and subset $G$ is the subset with the top $n_g$ good solutions. Fig. 2 shows the concept of goal softening at Stage 2. The alignment probability that the intersection between $G$ and $S$ has $k$ elements or more may be written as
\[
\text{prob}[G \cap S \geq k] = \frac{\min_{g \in G} \binom{N-g}{s-i}}{\binom{N}{s}}
\]

where
\[
\text{prob}[\cdot] \quad \text{alignment probability}
\]
- \(S\): subset of Set \(\Theta'\)
- \(s\): number of selected elements in subset \(S\)
- \(G\): subset with \(k\) or more good solutions in \(\Theta'\)
- \(|G\cap S|\): number of elements in the intersection between sets

Parameter \(s\) in subset \(S\) is determined by (7).

**Algorithm of Proposed Method**

Fig. 3 shows the concept of OO-PTS, where a part of solution candidates are randomly selected in the subneighborhood. OO is complementary used to evaluate enough good solutions in optimization problems. In this paper, a hybrid method of OO and PTS (OO-PTS) is proposed for transmission network expansion planning. Although TS examines all the solution candidates in the neighborhood, OO-PTS randomly selects a certain number of them. OO-PTS assumes that subset \(\Theta'\) of goal softening at Stage 1 corresponds to all the solution candidates in the neighborhood at each iteration. The algorithm of the proposed method may be summarized as follows:

1. **Step 1**: Set the initial conditions.
2. **Step 2**: Create the subneighborhoods with PTS and evaluate \(s\) solutions randomly selected from the subneighborhood, where a processor is assigned to each subneighborhood.
3. **Step 3**: Evaluate the solution at each subneighborhood.
4. **Step 4**: Select the best solution with the framework of tabu list.
5. **Step 5**: Stop if the termination conditions are satisfied. Otherwise, return to Step 2.

Fig. 4 gives the flowchart of the proposed method. OO-PTS carries out goal softening in the subneighborhood of PTS and evaluates the solution candidates of the predetermined number \(s\). That alleviates the computational effort for transmission network expansion planning.

**V. SIMULATION**

### A. Simulation Conditions

1. The proposed method is applied to the 46-node transmission system (see Fig. 5), where the system has 10 generator and 19 load nodes. The solid line denotes the existing lines. As the candidates of new line, 172 lines are assumed in the sample system.

2. The constraints in the formulation are given as follows:
   - Line flow limitation: \(P_i \leq 33.3\) [MW]
   - Maximum number of lines: \(n_{ij_{max}} = 7\)

Since the maximum number of lines \((n_{ij_{max}})\) is seven, each line is expressed in 3 bits. The total bits result in \(516(=3 \times 172)\). Thus, the number of combinations is about \(2.15 \times 10^{155}(=2^{516})\). That implies that the problem to be considered is very hard to solve.

3. The coding of line is expressed in binary number. Suppose that the solution candidates between nodes \(i\) and \(j\) is expressed as \(z_{ij}\). The variable may be written as

\[
z_{ij} = \begin{cases} 
0 & \text{(not installed)} \\
1 & \text{(installed)} 
\end{cases}
\]

### Method A: SA

### Method B: GA

### Method C: TS

### Method D: PTS

### Method E: OO-TS

### Method F: OO-PTS
To examine the influence of the initial conditions on the fine solution, fifty initial solutions are prepared to evaluate the reliability of the algorithm.

5) Table 2 shows the parameters of each method that are determined by the preliminarily simulation. The number of selected solutions (s) is set to be 85 for methods E and F. That implies that there is at least one solution in the top 25 solutions with probability 99% and more.

Table 2 Parameters of Each Method

<table>
<thead>
<tr>
<th>Methods</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Initial Temperature 200000</td>
</tr>
<tr>
<td></td>
<td>Cooling Schedule 0.99999</td>
</tr>
<tr>
<td></td>
<td>Convergence Criterion 5000</td>
</tr>
<tr>
<td>B</td>
<td>No. of Populations 500</td>
</tr>
<tr>
<td></td>
<td>No. of Generations 1000</td>
</tr>
<tr>
<td></td>
<td>Crossover Rate 0.8</td>
</tr>
<tr>
<td></td>
<td>Mutation Rate 0.01</td>
</tr>
<tr>
<td>C</td>
<td>Tabu Length 172</td>
</tr>
<tr>
<td></td>
<td>Maximum Iterations 1000</td>
</tr>
<tr>
<td>D</td>
<td>Tabu Length 165, 172</td>
</tr>
<tr>
<td></td>
<td>Maximum Iterations 1000</td>
</tr>
<tr>
<td>E</td>
<td>Tabu Length 170</td>
</tr>
<tr>
<td></td>
<td>Maximum Iterations 1000</td>
</tr>
<tr>
<td></td>
<td>N_P 85</td>
</tr>
<tr>
<td>F</td>
<td>Tabu Length 170, 165</td>
</tr>
<tr>
<td></td>
<td>Maximum Iterations 400</td>
</tr>
<tr>
<td></td>
<td>N_P 85</td>
</tr>
</tbody>
</table>

6) It is assumed that PTS has two subneighborhoods with two tabu lengths. Two tabu lengths are set to be 170 and 165.

7) The computation was performed on the Fujitsu Workstation S-7/400Ui m270D (Ultra-5, SPECinit 95: 9.1, and SPECfp 95: 10.1).

B. Simulation Results

Fig. 6 shows the cost function of each method, where the best, the average and the worst cost functions are given for the fifty initial conditions. Now, let us look at the best cost function.
Apart from Methods A and B, the other methods with TS found out the best solution. Also, looking at the worst function, we can see that Methods A and B are worse than others. Thus, Methods A and B are not as good as the TS-based methods. Methods C and D give the same best solution for the fifty initial solutions. Regarding Methods E and F, they were not so good as Methods C and E due to the use of OO. However, there is not significant difference between the TS-based and the OO-TS-based methods in terms of the average cost function. Methods E and F have about 4% errors for Methods C and E. The error is sufficiently acceptable in power system planning. Fig. 7 shows the standard deviation of the cost functions for each method. It can be seen that the trend of the performance is the same as the results of the average cost function. It can be seen that Methods C and D have the standard deviation of zero and Methods E and F are a little bit worse than Methods C and F. Compared with the results of Methods A and B, the TS-based methods are more reliable in a sense that they are not significantly influenced by the initial conditions. Fig. 5 shows the optimal network that corresponds to the best cost function. The dotted line denotes the installed new lines. Fig. 8 gives the average CPU time of each method. Method E is 12.78-times, 7.50-times, 4.48-times, and 2.61-times faster than Methods A, B, C, D, and E, respectively. Method D is about 4.48-times faster than Method C due to the use of PTS. Method F in which OO is applied to Method D is 4.48-times faster than Method C. That is because OO contributes to the calculation of solution candidates in the subneighborhood in PTS. Therefore, the proposed method allows the system planner to evaluate the transmission network expansion planning efficiently.

VI. CONCLUSION

(1) This paper has proposed a new method for transmission network expansion planning that is one of the most difficult combinatorial optimization problems in power systems. The proposed method makes use of a hybrid meta-heuristic method of ordinal optimization (OO) and parallel tabu search (PTS) to solve a combinatorial optimization problem. PTS is employed to improve the performance of TS in terms of solution accuracy and computational effort. PTS has strategies of the neighborhood decomposition and multiple tabu lengths. They contribute to the reduction of computational time and the diversity of the solution candidates. OO is introduced into PTS to reduce the computational effort of evaluating the solution candidates in the subneighborhoods in PTS.

(2) The proposed method was successfully applied to the 46-node system with about $2.15 \times 10^{35}$ solution candidates. To demonstrate the effectiveness of the proposed method, this paper made a comparison between the proposed and the conventional meta-heuristic methods such as SA, GA and TS. The proposed method gave a little bit worse solution with about 4% error, but it was 12.78-times, 7.50-times and 4.48-times faster than SA, GA and TS, respectively. The simulation results have shown that the proposed method is more applicable to real-size systems. Thus, the proposed method allows the system planner to carry out transmission expansion planning efficiently.

REFERENCES