Optimal Design of Grounding System in Transmission Line

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***Abstract*-- A novel optimization methodology is proposed for the design of transmission line grounding systems, taking into account technical as well as economical considerations. The problem of designing the grounding systems of transmission lines is stated as a linear-integer programming problem in terms of the construction characteristics and the particular requirements of the tower grounding schemes at the supports of each of the different line sections, in order to minimize the variable investment costs, subject to the maximum allowed line outage rate due to the lightning activity. The mathematical statement of the problem allows solutions in which the transmission tower footing resistance changes along the line, depending on the cost and on the particular characteristics of each tower grounding, assuring however, that the average behavior enforces the desired outage rate due to lightning activity, selecting the complementary electrode scheme required at each tower. The methodology is tested on a real case consisting of a 230 kV transmission line, 85.4 km long, with 180 towers. The linear programming branch and bound mathematical technique was applied for the solution of the test case. Two different simulation approaches for the calculation of the behavior of the line subject to lightning phenomena were evaluated without loss of generality: the approach proposed in [1], selected as an initial test due to its simplicity, and the improved version presented in [2]. Results are presented and compared to the design obtained through conventional tower design approaches with important reductions in the investment costs, encouraging the use and further development of the methodology.**

***Index Terms*--Transmission lines, grounding, lightning, optimization.**

I. Nomenclature

$a_{i,j}$ counterpoise conductor length (m) of complementary electrode scheme j at line section S_i

a^i_{mn} distance between conductor m and the image of conductor n, at line section S_i (m)

B_i distance between the shield wires at line section S_i (m)

$b_{i,j}$ soil excavation, filling and compacting volume (m^3) of complementary electrode scheme j at line section S_i

b^i_{mn} distance between conductor m and conductor n, at line section S_i (m)

$b_{\max}$ maximum number of cubic meters of excavation (m^3)

C thermal coefficient of air expansion, (C=0.003671/C°)

$c_{i,j}$ number of rods or bars of complementary electrode scheme j at line section S_i

C_A, C_B, C_C, C_D counterpoise length in directions A, B, C, D (m)

D_A, D_B, D_C, D_D maximum counterpoise length in directions A, B, C, D (m)

G_{ig} set of distances soil layers depths at line section S_i (m)

$d_{i,j}$ drilling or perforation length (m) of complementary electrode scheme j at line section S_i

$e_{i,j}$ complementary electrode scheme j at line section S_i

E_i set of feasible complementary electrodes associated to line section S_i

g_{ig} depth of first soil layer at line section S_i (m)

H altitude above the sea level (m)

h_i effective height of the shield wires above the ground level at line section S_i (m)

h^i_n height of conductor n, at line section S_i (m)

K keraunic level of the geographical area, in thunderstorm days per year

I lightning stroke current (kA)

I_i current on conductor i (kA)

I^i_c critical flashover current of line section S_i (kA)

I_T tower current (kA)

k^i_1 coupling coefficient with phase 1

l length of conductor section (m)

L_i length of line section S_i (km)

L_T tower inductance (H)

L_L total length of the transmission line (km)

n total number of line sections of the line,

N_a number of sections of counterpoise C_A

N_b number of sections of counterpoise C_B

N_c number of sections of counterpoise C_C

N_d number of sections of counterpoise C_D

N_{Ci} probability of the direct lightning strokes to the transmission line that exceeds the value of the critical flashover current I_{Ci} at line section S_i

N_g Lightning ground-flash density (km^{-2} year^{-2})

N_i number of towers at line section S_i

N_{Li} number of direct lightning strokes at line section S_i for each 100 km per year

P atmospheric pressure (mmHg)

P_i total number of feasible complementary electrodes of line section S_i

P_o atmospheric pressure at sea level (760mmHg)

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Rg_{ij} grounding impedance of the supports of line section S_i with complementary grounding electrode scheme $e_{i,j}$, in impulse regime or dynamic conditions (Ω)
R_{ij} static or steady state grounding resistance of towers of section S_i with complementary electrode scheme $e_{i,j}$ (Ω)
r_n radio of conductor n (m)
S_i line section i
T_a air temperature (°C)
T forced outage rate of the transmission line for each 100 km per year
T_i number of insulation failures of line section S_i due to back-flashover associated to lightning phenomena for each 100 km per year
T_{ij} number of back-flashover insulation failures due to lightning phenomena of line section S_i with complementary electrode scheme $e_{i,j}$, for each 100 km per year
V_s disruptive voltage of the insulator chain due to impulse surges (kV)
V_{sn} critical disruptive voltage of the insulator chain under impulse surges for standard test conditions (kV)
$Z^i{}_{mn}$ mutual impedance between conductors m and n at line section S_i (Ω)
$x_{i,j}$ integer decision variable associated to the installation of complementary electrode scheme $e_{i,j}$ in the supports of line section S_i
α conductor cost ($/m)
β_ι excavation and filling cost at line section S_i ($/m^3)
γ cost of the rods or bars ($/unit)
η_i perforation or drilling cost at line section S_i ($/m)
$\rho_{\iota,1}$ first layer resistivity at line section S_i (Ω-m)
$\rho_{\iota,2}$ second layer resistivity at line section S_i (Ω-m)

II. Introduction

The design of the grounding system of transmission lines has an important impact on number of outages due to lightning strokes. It has been traditionally performed using well-known techniques documented in the literature. These techniques are aimed to enforce the electrical behavior of the insulation of the transmission line in transient conditions due to the lightning activity. In particular, the widely used methodologies proposed in [1] and [2], permit the calculation of the forced outage rate of the transmission line for the given keraunic levels of the geographical area, and the transmission line characteristics, such as the geometrical disposition of the conductors in space as well as the insulation level and the average equivalent grounding resistances of the different transmission line sections. Using the desired admissible value for the average grounding resistance, then the specific grounding schemes are specified for each of the towers of the different line sections, according to the physical characteristics of each of the tower locations, without the application of any mathematical optimization process. Other design methodologies have been proposed that improve the modeling and the design of the grounding system of transmission and distribution lines [3], [4], [5], [6] and further enhancements may be developed, however, the use of variations of this approach is a common practice among engineers world wide.

Considerable effort has been devoted aiming to the reduction of the grounding resistance of transmission lines [5], [6]. Different designs have been proposed for the complementary electrode schemes to be installed at the transmission tower supports [7], [8]. The behavior of the metallic structure of the towers of the transmission lines has also been studied [9]. These methodologies are based on the assumption that the grounding system must fulfill the requirement of an admissible equivalent grounding resistance value at each support, corresponding to a desired forced outage rate value; however, they do not take into account explicitly the cost of the grounding system and therefore the existence of better solutions with reduced costs is an open possibility.

The civil works required in order to comply with the required behavior of the grounding system are important and have a considerable ecological as well as economic impact on the construction of the project.

In this work, a technical and economical mathematical model is proposed for the minimization of the total investment costs of the required complementary electrodes of all the towers of the transmission line, enforcing as constraints: the outage rate of the electric line due to the transient overvoltage phenomena that takes place after lightning strokes, as well as the physical space availability for the placement of the complementary electrodes during the grounding construction of each of the tower foundations and the ecological limitations on the total volume of soil excavation for each of the tower locations.

The cost the grounding installation depends upon the characteristics of the soil at each of the tower locations, consequently the methodology allows the variation of the number of required complementary electrodes, and therefore of the equivalent grounding resistance, according to the construction costs at each location, but assuring the overall behavior of the transmission line, this is the desired forced outage rate.

The proposed methodology does not impose an equivalent grounding resistance value at each of the towers, instead it enforces the overall forced outage rate of the of the transmission line allowing the variation of the grounding resistance throughout the line.

The optimization problem is solved using the conventional branch and bound integer-linear programming approach.

The proposed optimization approach is applied to a real case consisting of a 230 kV transmission line with a total of 180 towers and eight line sections using both simulation methodologies: [1] and [2]. Results are presented using a set of 11 types of feasible grounding complementary electrode schemes with important reductions in the investment costs of 22% and 23% respectively, when compared to the design obtained through a conventional uniform design approach. These cost reductions were 8% and 6% when compared with the results obtained by applying the section-by-section grounding selection approach. Optimization results are also presented for the application of the simulation methodology [2], but incrementing the number of feasible types of grounding complementary electrode schemes to 3102, obtaining important savings in the order of 38% in the investment costs when compared to the design obtained through a conventional uniform design approach, and to 12%

when the selection is performed section by section using the average required behavior of the line. The number of decision variables that were present in the formulation of the optimization problem increased from 88 to 24816 binary variables after the set of feasible complementary grounding electrodes was augmented.

The results highly encourage the use and further development of the proposed methodology.

III. Statement Of The Problem

The design problem of the transmission line grounding system may be stated as an optimization problem, and furthermore as a integer-linear programming problem, as follows: minimize the investment cost of the grounding system, subject to the maximum limit on the forced outage rate of the transmission line due to lightning strokes, and to the ecological constraint of the maximum excavation volume permitted.

A. Cost function

The objective function to be minimized consists of a linear function that describes the variable components of the investment costs as the sum of the investment in counterpoises plus the investment in rods. The fixed cost components are not included in the formulation since they can not be modified. These variable cost components comprise the cost of the materials as well as the cost of the construction and other activities associated to the installation of the counterpoises and rods, at each of the tower locations.

The cost of the counterpoises must take into account the cost of the material, the cost of deforestation, of the exothermic connections, of the excavation and of the compacting of the soil.

The investment cost of the rods must consider the unitary costs of the material as well as the cost of the drilling. The excavation or digging and drilling costs depend upon the type of soil at each of the locations.

B. Model for the determination of the line outage rate due to lightning strokes

The mathematical models presented in [1] and [2], for the calculation of the line outage rate due to the lightning activity were used in this work, and the results were compared. Some additional variables have been included in order to allow the formulation of the optimization problem. The line outages due to shielding failures were neglected, assuming that the line is properly shielded.

It is assumed that the transmission line under study consists of a total of n different line sections, according to the characteristics of the ground, to the atmospheric activity and to the dominant type of tower supports that are present. The set of line sections is described as:

$$\{S_1,...,S_i,...,S_n\} \quad (1)$$

associatted to a set of line section lengths:

$$\{L_1, L_2,...,L_i,...,L_n\} \quad (2)$$

For each line section S_i the values of the soil resistivity model are calculated. If the bi-stratified or two layer model is applied, then for each line section i a two element resistivity vector RVD_i and the distance between layers or thickness G_i are calculated:

$$RVD_i = \{\rho_{i,1}, \rho_{i,2}\};\ G_i = \{g_{i,1}\} \quad (3)$$

If a multilayer model is used, then the dimensions of the vectors RVD_i and H_i are $n+1$ and n respectively:

$$RVD_i = \{\rho_{i,1}, \rho_{i,2},...\rho_{i,n+1}\};\ G_i = \{g_{i,1},...,g_{i,n}\} \quad (4)$$

A dominant or more frequent type of tower support must be defined for each line section S_i. The geometrical disposition in space of the phase conductors and of the shield wires for the dominant tower support is required for the calculations, for each line section S_i, as illustrated by Fig.1.

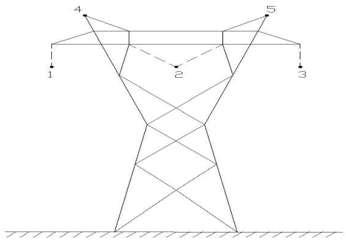

Fig. 1. Geometry of the line sections

It is assumed that the outage rate of the line due to direct lightning strokes to the phase conductors is negligible.

Self and mutual inductances need to be calculated, since they are required for the determination of the outage rate of the line due to the back-flashover phenomena caused by lightning strokes on the shield wires.

The required impedances for the calculation of the outage rate of phase 1 of the line due to the back-flashover phenomena, to be calculated for each line section i using equations (5),(6) are Z^i_{11}, Z^i_{14}, Z^i_{15}, Z^i_{44}, Z^i_{45} y Z^i_{55} [1] :

$$Z^i{}_{nn} = 60 \cdot \ln(2h^i{}_n / r_n) \quad (5)$$

$$Z^i{}_{mn} = 60 \cdot \ln(a^i{}_{mn} / b^i{}_{mn}) \quad (6)$$

Fig. 2 illustrates the definitions of: h_n, b_{mn}, a_{mn}. The superscript i indicates the correspondent line section S_i.

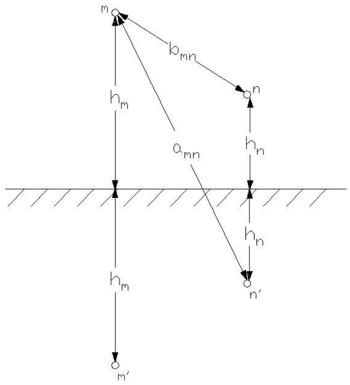

Fig. 2. Line dimensions for impedance calculations

In order to determine the outage rate T, the critical flashover current at each line section must be calculated based on the equivalent circuit shown in Fig.3. According to [1] and after some algebraic operations the following expression for the critical flashover current $I^i{}_c$ is obtained:

$$I^i{}_c = V_s(Z^i{}_{44} + Z^i{}_{45} + 4R_{g_{ij}})/((1-k^i{}_1)(Z^i{}_{44} + Z^i{}_{45})R_{g_{ij}}) \quad (7)$$

where $k^i{}_1$ is the coupling coefficient with phase 1 defined as:

$$k^i{}_1 = (Z^i{}_{14} + Z^i{}_{15})/(Z^i{}_{54} + Z^i{}_{55}) \tag{8}$$

The approach recommended in [2] may be used instead if a more precise calculation is required for the critical flashover current. In this case, the effective ratios of the shield and phase wires considering the corona effect are calculated, before the calculation of the coupling coefficients, which are determined for all the phases in order to choose the worst case. The response of the transmission tower to the lightning flash is computed in a more elaborated manner, calculating the tower top voltage, the cross arm voltage, the insulator surge voltage, the reflection from adjacent towers and the effects of the power frequency voltage [2].

The grounding impedance in impulse regime, i.e., the equivalent impedance of the grounding scheme in presence of very fast transient conditions and high currents or dynamic grounding resistance Rg_{ij} of the tower supports at line section S_i with complementary electrode scheme $e_{i,j}$, is related to the grounding resistance in low current and static conditions or steady state R_{ij} [1][2]. In this work, the following simplistic relation between Rg_{ij} and R_{ij} was used, without loss of generality for the application of the proposed methodology:

$$Rg_{ij} = R_{ij}/4 \tag{9}$$

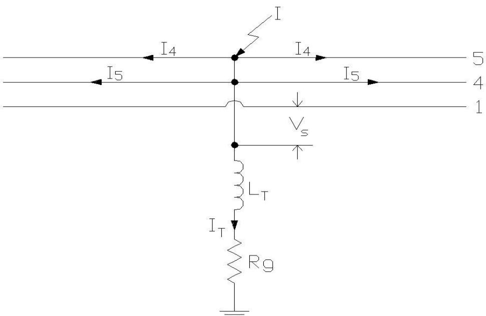

Fig. 3. Equivalent circuit for back-flashover analysis at phase 1

It is patent that the relation between R_{ij} and Rg_{ij} is very complex and that expression (9) represents and approximation which may be subject to discussion. It must be clearly stated that the simplified expression used by the authors for the impulse impedances in terms of the power frequency impedances does not represent a limitation for the proposed optimization methodology, since any other relation may be used instead, prior to the application of the optimization approach, at the stage of the formulation of the optimization problem. In [2] the use of the plot presented in [14] of the impulse resistance as a function of the power frequency resistance is suggested. In a more accurate approach, for example, the transient electromagnetic phenomena that take place after a lightning stroke can be simulated using specialized software, to calculate an equivalent grounding resistance of each of the corresponding grounding schemes [5].

The value of V_s must be determined for the particular climatic conditions of the geographical area under study [15]:

$$V_s = 0.38\, V_{sn} P/(273 + T_a) \tag{10}$$

where the atmospheric pressure P was calculated using the following expression [17]:

$$P = (P_o / 10^{\left(\frac{H}{18400\,(1+C\cdot Ta)}\right)}) \tag{11}$$

The forced outage rate of the transmission line is calculated by the following expression:

$$T = \left[\sum_{i=1}^{n} T_i L_i\right] / L_T \tag{12}$$

For each section S_i of the line the variable T_i is determined. It represents the number of insulation failures due to back-flashover associated to lightning phenomena of section L_i of line section S_i for each 100 km per year [1], [2]:

$$T_i = 0.6 N_{L_i} N_{C_i} \tag{13}$$

Although the use of a factor of 0.6 may be arguable, this value was left in order to follow the well known approaches suggested by [1] or [2]. The variable N_{Li} represents the number of direct lightning strokes received by the transmission line for each 100 km per year. If measurements of the atmospheric activity are not available, alternatively, the number of thunderstorm days per year may be used to estimate the value of N_{Li} using (14), with h_i and B_i defined as in Fig.4, [2] although more precise expressions such as those proposed in [10] may be used instead.

$$N_{Li} = 0.1 \cdot N_g \cdot (4h_i^{1.09} + B_i) \tag{14}$$

where N_g was set according to [16]:

$$N_g = 0.04 \cdot K^{1.25} \tag{15}$$

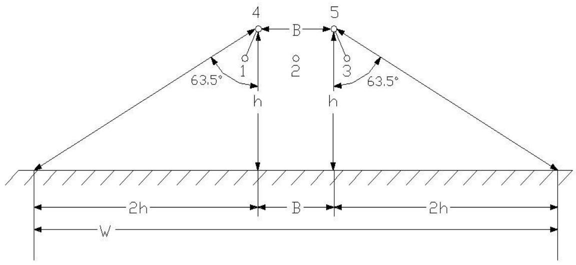

Fig. 4. Conductor positions for lightning protection area calculation

N_{Ci} is defined as the probability of the direct lightning strokes to the transmission line that exceeds the value of the critical flashover current $I^i{}_C$ at line section S_i. It is assumed that N_{Ci} follows a probabilistic distribution [2]:

$$N_{Ci} = 1/\left[1 + (I_C^i/31)^{2.6}\right] \tag{16}$$

C. Generation of complementary grounding electrode schemes

Each of the tower supports of line section S_i under study has a basic grounding resistance R_{Ti} corresponding to the effect of the metallic structures buried in the ground. The value of R_{Ti} depends on the characteristics of the ground and on the particular dimensions of each tower support. It is possible to obtain typical values of R_{Ti}, on the basis of field tests or by widely used finite element simulation methods [9], [11], [12].

The achievement of better grounding resistance levels, aiming to the reduction of the line outage rate, leads to the installation of complementary grounding electrodes, based on counterpoises and/or rods, as shown in Fig. 5 [7][8]. The geometry of the complementary electrode schemes depends on the available space in the surroundings of the respective tower, and on the grounding requirements of the line.

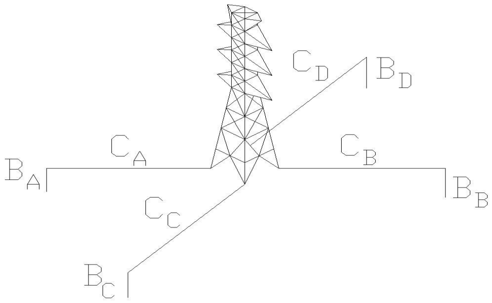

Fig. 5. Complementary electrode scheme of a tower support

The methodology generates a number of feasible complementary electrodes P_i for each line section S_i.

D. Complementary electrode schemes

Different grounding complementary electrode schemes may be constructed combining counterpoises and rods. Fig.6 illustrates some possible configurations of the complementary electrode schemes. The set E_i of complementary electrode schemes associated to line section S_i is defined as:

$$E_i = \{e_{i,1}, e_{i,2}, ..., e_{i,j}, ..., e_{i,P_i}\} \tag{17}$$

The counterpoises of complementary electrode schemes $e_{i,j}$, are placed successively in a maximum of four (4) perpendicular directions: A, B, C and D. The length of each counterpoise must accomplish the space constraints established by parameters D_A, D_B, D_C, D_D (maximum lengths in directions A, B, C and D).

$$\begin{aligned} C_{A_{i,j}} &= N_{a_{i,j}} l \le D_{A_{i,j}} \\ C_{B_{i,j}} &= N_{b_{i,j}} l \le D_{B_{i,j}} \\ C_{C_{i,j}} &= N_{c_{i,j}} l \le D_{C_{i,j}} \\ C_{D_{i,j}} &= N_{d_{i,j}} l \le D_{D_{i,j}} \end{aligned} \tag{18}$$

The complementary electrode of type (6-10) has rods at the end of each counterpoise. Each rod is 3.3 m long and the parameters B_A, B_B, B_C, B_D indicate the number of rods installed at the end of each counterpoise. The rods or bars come in standard dimensions (10 ft. or 3.3 m) and are installed one over the other after the drilling of the soil, producing an equivalent rod of higher length.

The total number of considered electrode schemes is determined from the assignment of discrete values to the variables l, b_a, b_b, b_c, b_d. The combination of all the pre-established arrangements (numbered from 1 to 10) is performed by the algorithm enforcing the space limitations constraints for the supports of line section $s_{i.}$

Once the geometry of each generated electrode scheme $e_{i,j}$ has been defined, the required amounts $a_{i,j}$, $b_{i,j}$, $c_{i,j}$ and $d_{i,j}$ of construction materials (counterpoises and rods), excavation, filling and compacting volumes, drilling meters as well as the grounding resistance $R_{i,j}$, are calculated for the supports of each line section S_i, considering its particular resistivity conditions.

E. Calculation of the required amounts of construction materials

The following calculations are required in order to determine the cost of each grounding scheme:

i) Counterpoise conductor length (in meters):

$$a_{i,j} = C_{A_{i,j}} + C_{B_{i,j}} + C_{C_{i,j}} + C_{D_{i,j}} \tag{19}$$

ii) Soil excavation, filling and compacting volume (in cubic meters, for 0.50 m wide and 0.50 m depth excavations):

$$b_{i,j} = a_{i,j} / 4 \tag{20}$$

iii) Number of rods or bars:

$$c_{i,j} = B_{A_{i,j}} + B_{B_{i,j}} + B_{C_{i,j}} + B_{D_{i,j}} \tag{21}$$

iv) Drilling or Perforation length (in meters):

$$d_{i,j} = 3.3\, c_{i,j} \tag{22}$$

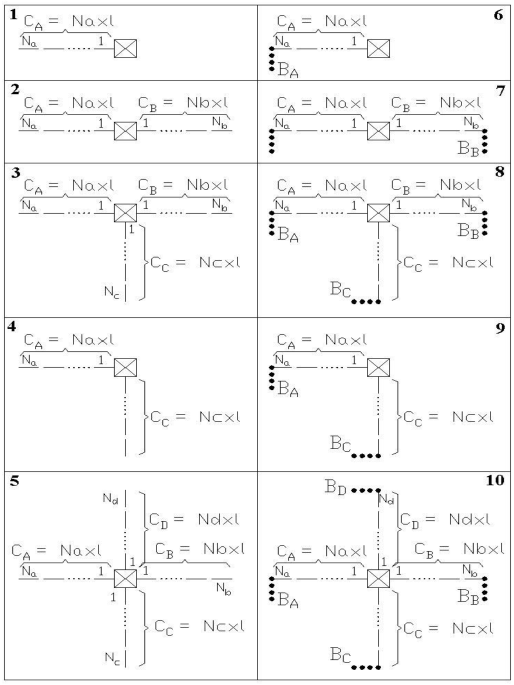

Fig. 6. Basic Complementary Grounding Electrode Schemes

F. Calculation of equivalent grounding resistance R_{IJ}

The calculation of the equivalent static or steady state grounding resistance R_{ij} of the supports of line section S_i for each of the feasible complementary electrode schemes $e_{i,j}$ that were generated, is performed by means of the image method. This methodology can be applied to a multiple layer soil [12] considering the self resistance R_{Ti}. [9]

G. Linear programming formulation

Formulated as such, the problem can be stated as an integer-linear program:

H. Objective function

The objective is the minimization of a linear function in terms of the number and location of the grounding electrodes that are to be installed, considering the conductor cost, the excavation and drilling costs and the cost of the rods, as follows:

$$min \sum_{i=1}^{n} N_i \left\{ \sum_{j=1}^{P_i} \alpha a_{i,j} x_{i,j} + \beta_i b_{i,j} x_{i,j} + \gamma c_{i,j} x_{i,j} + \eta_i d_{i,j} x_{i,j} \right\} \tag{23}$$

I. Constraints

i) Single grounding scheme selection constraint. For each of the tower supports of line section S_i only one electrode scheme $e_{i,j}$ must exist:

$$\sum_{j=1}^{P_i} x_{i,j} = 1 \qquad i = 1,\dots,n \tag{24}$$

ii) Maximum Forced Outage Rate.
This constraint enforces the overall lightning behavior of the transmission line:

$$\left(\sum_{i=1}^{n}\sum_{j=1}^{P_i} L_i T_{i,j} x_{i,j}\right) / L_L \leq T \tag{25}$$

where $T_{i,j}$ is calculated a priory.
If approach proposed in [1] is applied then $T_{i,j}$ is defined by :

$$T_{i,j} = 0.0024 \cdot K^{1.25} \cdot (4h_i^{1.09} + B_i) / \left(1 + \left(I_c^i / 31\right)^{2.6}\right) \tag{26}$$

If $T_{i,j}$ is calculated using [2] instead, the effective ratios of the shield and phase wires considering the corona effect need to be calculated before the computation of the coupling coefficients, which are calculated for all the phases in order to choose the worst case (instead using only the most distant phase as in [1]). The response of the transmission tower to the lightning flash is computed in a more elaborated manner, calculating the tower top voltage, the cross arm voltage, the insulator surge voltage, the reflection from adjacent towers and the effects of the power frequency voltage [2].
It is important to mention that within the scope of the proposed design approach, more precise, simulation approaches may be used to determine $T_{i,j}$ prior to the solution of the optimization problem.
iii) Excavation volume limits. In some particular cases, ecological constraints may exist, defining a maximum limit on the soil volume to be removed, in cubic meters.

$$\sum_{i=1}^{n}\sum_{j=1}^{P_i} N_i b_{i,j} x_{i,j} = \quad b_{\max} \tag{27}$$

IV. Proposed Methodology

The integer linear programming problem stated in the previous section can be solved using a number of well known and widely used techniques. In this work, the Revised Simplex Method was applied using the linear programming branch and bound technique for handling the integer variable constraints [13].

V. Results

The grounding system of a test case was designed using the proposed optimization approach and the two different simulation methodologies presented in [1] and [2]. The results are compared with those obtained using a section by section approach, where the grounding electrode scheme selection of each line section is designed to enforced the desired maximum outage rate without taking into account the behavior of the rest of the line sections. The results are also compared with those obtained by selecting a single standard complementary electrode for all the supports of the line.

A. Test Case:

The test case consists of an 85.4 km long, single circuit 230 kV transmission line with 180 towers and two shield wires (conductor type ACSR 211.3). The insulation has a Critical Flashover Voltage (CFO) of 1390 kV. An average number of 76 thunderstorm days/year was used in the calculations. The desired forced outage rate of the line was set to 0.5 outages per 100 km per year. The effect of the self resistance of the tower foundations without the complementary electrodes (R_{Ti}) was neglected. There were no ecological constraints considered. The cost coefficients for the calculation of the objective function where: α = 20 \$/m; β = 100 \$/m^3; γ = 20 \$/unit. The excavation costs were assumed to be a function of the resistivity of the two soil layers at the particular locations of the supports for each line section:

$$\eta_\iota = 100\left(\left(g_{ig}\,\rho_{i,2}/\rho_{i,1}\right) + 3\right) \tag{28}$$

The characteristics of the line towers are presented in Tables I and II. The set of feasible complementary electrode schemes composed by 11 different types of electrodes, resulted to be the same for all the tower supports. It is presented in Table III.

B. Results:

Results are presented of the application of three different approaches using simulation methodologies [1] and [2]:

1. the proposed optimization approach,
2. the section by section grounding scheme selection,
3. the selection of a single, uniform, grounding scheme for the whole transmission line.

C. Simplified simulation methodology [1]

Initially, and without loss of generality, the simulation methodology proposed in [1] was selected as an initial test due to its simplicity. The optimization problem was stated as the minimization of linear objective function with 88 decision variables subject to 8 equality constraints and one inequality constraint. The resultant complementary electrode schemes selected by the proposed optimization methodology, as well as the equivalent grounding resistances are shown in Table IV.

Table V presents the selected complementary grounding electrode schemes, as well as the equivalent grounding resistance after the application of the section by section selection approach and the simulation methodology proposed in [1], where the construction designs of each of the grounding schemes are enforced to produce an admissible value for the equivalent grounding resistance (less than or equal to the average grounding resistance value determined by the methodology presented in [2]), according to the desired forced outage rate of the line.

Table VI shows the results of selecting a single standard grounding complementary electrode for all the towers of the transmission line. Grounding scheme e_{10} was selected (two counterpoises of 100m with two rods of 3.3 m each), since it is the lowest cost complementary electrode scheme that accomplishes the maximum outage rate constraint.

Table VII presents the variable construction costs for the grounding schemes obtained by each of the three selection approaches.

TABLE I
CHARACTERISTICS OF THE LINE

SECTION	NUMBER OF TOWERS	L (m)	SOIL PARAMETERS ρ_1 (Ω-m)	ρ_2 (Ω-m)	g (m)
S_1	30	16380.0	3000.0	500.0	3.0
S_2	18	7140.0	350.0	550.0	2.5
S_3	16	7980.0	2550.0	350.0	1.3
S_4	22	9660.0	5000.0	100.0	2.5
S_5	24	9660.0	2750.0	350.0	2.5
S_6	22	8400.0	5300.0	500.0	4.0
S_7	18	10080.0	500.0	250.0	3.0
S_8	30	16100.0	4800.0	150.0	3.0
TOTAL	180	85400.0			

TABLE II
CHARACTERISTICS OF THE LINE TOWERS

TYPE OF TOWER	1	2	3	4	5
$r_4 = r_5$ (m)	0.008435	0.008435	0.008435	0.008435	0.008435
B (m)	13.4	9.75	16	11.9	16
a_{14} (m)	89.7	59.52	70.34	81.27	80.11
a_{15} (m)	88.7	58.72	68.5	80.7	78.5
a_{45} (m)	108.23	78.17	99.29	112.1	109.17
b_{14} (m)	23	21.21	33.55	22.16	33.55
b_{15} (m)	18.7	18.84	29.5	18.7	29.5
b_{45} (m)	13.4	9.75	16	11.9	16
h (m)	53.7	38.5	49	50.1	54

TABLE III
SET OF FEASIBLE GROUNDING ELECTRODE SCHEMES

ELECTRODE	COUNTERPOISE (m)				NUMBER OF RODS			
$e_{i,j}$	C_A	C_B	C_C	C_D	B_A	B_B	B_C	B_D
j=1	100	0	0	0	0	0	0	0
j=2	0	0	0	0	1	0	0	0
j=3	0	0	0	0	2	0	0	0
j=4	100	0	100	0	0	0	0	0
j=5	100	100	100	100	0	0	0	0
j=6	100	0	0	0	1	0	0	0
j=7	100	0	100	0	1	0	1	0
j=8	100	100	100	100	1	1	1	1
j=9	100	0	0	0	2	0	0	0
j=10	100	0	100	0	2	0	2	0
j=11	100	100	100	100	2	2	2	2

TABLE IV
OPTIMIZATION RESULTS WITH A REDUCED SET OF FEASIBLE GROUNDING ELECTRODES AND SIMULATION METHODOLOGY [1]

SECTION	ELECTRODES $e_{i,1}$	$e_{i,2}$	$e_{i,3}$	$e_{i,4}$	$e_{i,5}$	$e_{i,6}$	$e_{i,7}$	$e_{i,8}$	$e_{i,9}$	$e_{i,10}$	$e_{i,11}$	R (Ohms)	T (T/100Km/year)
S_1				■								15.15	0.770
S_2	■											7.68	0.133
S_3				■								9.17	0.210
S_4										■		8.48	0.172
S_5							■					11.97	0.419
S_6					■							3.79	0.021
S_7	■											6.48	0.085
S_8										■		15.98	0.884
FORCED OUTAGE RATE OF TRANSMISSION LINE													0.424

The savings obtained by the optimization approach are explained due to the relaxation of the outage rate constraint in those sections where the grounding construction is more expensive (sections 1 and 8 of the line), and compensated by more investment in those areas where the construction is less expensive, reducing the number of insulation failures in the latter ones, assuring the overall behavior of the whole line.

TABLE V
RESULTS OF THE SECTION BY SECTION GROUNDING SCHEME SELECTION WITH A REDUCED SET OF FEASIBLE GROUNDING ELECTRODES AND SIMULATION METHODOLOGY [1]

SECTION	ELECTRODES $e_{i,1}$	$e_{i,2}$	$e_{i,3}$	$e_{i,4}$	$e_{i,5}$	$e_{i,6}$	$e_{i,7}$	$e_{i,8}$	$e_{i,9}$	$e_{i,10}$	$e_{i,11}$	R (Ohms)	T (T/100Km/year)
S_1										■		12.80	0.498
S_2	■											7.68	0.133
S_3				■								9.17	0.210
S_4										■		8.48	0.172
S_5				■								12.77	0.495
S_6	■											9.17	0.204
S_7	■											6.48	0.085
S_8											■	8.68	0.182
FORCED OUTAGE RATE OF TRANSMISSION LINE													0.258

TABLE VI
RESULTS OF THE SELECTION OF A SINGLE GROUNDING COMPLEMENTARY ELECTRODE SCHEME WITH A REDUCED SET OF FEASIBLE GROUNDING ELECTRODES AND SIMULATION METHODOLOGY PROPOSED IN [1]

SECTION	ELECTRODES $e_{i,1}$	$e_{i,2}$	$e_{i,3}$	$e_{i,4}$	$e_{i,5}$	$e_{i,6}$	$e_{i,7}$	$e_{i,8}$	$e_{i,9}$	$e_{i,10}$	$e_{i,11}$	R (Ohms)	T (T/100Km/year)
S_1										■		12.80	0.498
S_2										■		4.33	0.030
S_3										■		7.71	0.134
S_4										■		8.48	0.172
S_5										■		10.21	0.278
S_6										■		5.41	0.054
S_7										■		3.31	0.015
S_8										■		15.98	0.884
FORCED OUTAGE RATE OF TRANSMISSION LINE													0.335

It can be observed that the differences in the construction design are traduced in estimated savings in the order of 8% and 24% in the variable components of the investment costs, with respect to the results obtained using the section by section design approach or the single standard complementary grounding electrode scheme, respectively.

TABLE VII
RESULTS OF THE THREE DESIGN APPPROACHES WITH A REDUCED SET OF FEASIBLE GROUNDING ELECTRODES AND THE SIMULATION METHODOLOGY PROPOSED IN [1]

	DESIGN METHODOLOGY BASED IN [1]		
	(1) PROPOSED	(2) SECTION BY SECTION	(3) SINGLE SCHEME
SECTION	COST ($)	COST ($)	COST ($)
S_1	270,000	399,000	399,000
S_2	81,000	81,000	314,000
S_3	144,000	144,000	207,000
S_4	281,000	281,000	280,000
S_5	265,000	216,000	314,000
S_6	396,000	99,000	289,000
S_7	81,000	81,000	261,000
S_8	384,000	768,000	384,000
OBJECTIVE	**1,902,000**	**2,069,000**	**2,448,000**

D. Simulation methodology proposed in [2]

The results of the optimization are shown in Tables VIII and IX when the simulation methodology proposed in [2] was used with a set of 11 feasible grounding electrode schemes for both approaches: the proposed optimization methodology and the section by section selection approach.

If an uniform grounding scheme approach is applied, the selected grounding scheme is similar to that reported previously in Table VI, but the corresponding outage rate is slightly higher 0.43 outages/100 km per year.

Table X presents a comparison of the results of the three methodologies, showing the resultant costs of the selected complementary grounding electrode schemes for each of the sections.

TABLE VIII
OPTIMIZATION RESULTS WITH A REDUCED SET OF FEASIBLE GROUNDING ELECTRODES AND SIMULATION METHODOLOGY PROPOSED IN [2]

SECTION	ELECTRODES											R	T
	$e_{i,1}$	$e_{i,2}$	$e_{i,3}$	$e_{i,4}$	$e_{i,5}$	$e_{i,6}$	$e_{i,7}$	$e_{i,8}$	$e_{i,9}$	$e_{i,10}$	$e_{i,11}$	(Ohms)	(T/100Km/year)
S_1										■		12.80	0.560
S_2	■											7.68	0.157
S_3				■								9.17	0.359
S_4									■			15.96	0.829
S_5				■								14.18	0.624
S_6					■							3.79	0.799
S_7	■											6.48	0.112
S_8										■		15.98	0.388
FORCED OUTAGE RATE OF TRANSMISSION LINE													0.483

TABLE IX
RESULTS OF THE SECTION BY SECTION DESIGN APPROACH WITH A REDUCED SET OF FEASIBLE GROUNDING ELECTRODES AND SIMULATION METHODOLOGY PROPOSED IN [2]

SECTION	ELECTRODES											R	T
	$e_{i,1}$	$e_{i,2}$	$e_{i,3}$	$e_{i,4}$	$e_{i,5}$	$e_{i,6}$	$e_{i,7}$	$e_{i,8}$	$e_{i,9}$	$e_{i,10}$	$e_{i,11}$	(Ohms)	(T/100Km/year)
S_1					■							12.80	0.210
S_2	■											7.68	0.157
S_3				■								9.17	0.359
S_4										■		8.48	0.163
S_5							■					12.77	0.490
S_6				■								11.72	0.391
S_7	■											6.48	0.112
S_8										■		15.68	0.388
FORCED OUTAGE RATE OF TRANSMISSION LINE													0.286

TABLE X
COMPARISON OF THE RESULTS OF THE THREE DESIGN APPROACHES WITH A REDUCED SET OF FEASIBLE GROUNDING ELECTRODES AND SIMULATION METHODOLOGY PROPOSED IN [2]

	DESIGN METHODOLOGY BASED IN [2]		
	(1) PROPOSED	(2) SECTION BY SECTION	(3) SINGLE SCHEME
SECTION	COST ($)	COST ($)	COST ($)
S_1	411,000	540,000	399,000
S_2	81,000	81,000	314,000
S_3	144,000	144,000	207,000
S_4	144,000	289,000	280,000
S_5	216,000	270,000	314,000
S_6	396,000	190,000	289,000
S_7	81,000	81,000	261,000
S_8	395,000	395,000	384,000
OBJECTIVE	**1,868,000**	**1,990,000**	**2,448,000**

C. Augmentation of the set of feasible grounding electrodes

In order to obtain more realistic results, the number of feasible dimensions of the counterpoises was increased, resulting in an increased number of feasible types of complementary grounding electrodes (3102).

In this case the linear integer programming problem consists of 24816 variables.

It can be observed that the objective function of the optimization with 3102 feasible schemes (US$1.547.000) is 17% less with respect to the previous optimization performed with only 11 feasible schemes (US$1.868.000). Savings of 12.4% with respect to the section by section methodology (US$1.770.000) are obtained. The savings increase to 37.6% when compared to the single grounding scheme results (US$2.448.000).

V. CONCLUSIONS

A new methodology is presented for the design of transmission line grounding systems. The problem is stated as an integer-linear programming optimization problem and solved using the branch and bound technique and the Conventional Revised Simplex Method for linear programming. The methodology selects the set of complementary grounding electrode schemes that are required to comply with the average forced outage rate constraint at a minimum investment cost, considering the amount of material used in the construction, the deforestation costs, the soil excavation, filling and compacting costs, the drilling costs, as well as the material availability constraints, ecological constraints, and space availability constraints.

The methodology is applied to a real case consisting of a 230 kV transmission line section with 180 towers, and the results were compared with those obtained using a conventional technique for the complementary grounding electrode design that enforces a constant value for the equivalent grounding resistance throughout the line.

Results reflect important savings in the investment costs, enforcing the given forced outage rate value of the transmission line.

Among the contributions of this article is the design of transmission line grounding systems allowing the outage rate of some sections to exceed the maximum value, due to the high cost of the corresponding grounding system, and compensating the overall behavior of the line due to lightning strokes with the grounding of those sections with smaller costs.

The technique is shown as a valuable tool for the design of transmission line grounding systems.

VI. ACKNOWLEDGMENT

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VI. REFERENCES

[1] Electric Power Research Institute (EPRI), *Transmission Line Reference Book, 345 kV and above, Chapter 12,* First Edition, Palo Alto California, 1975.

[2] Electric Power Research Institute (EPRI), *Transmission Line Reference Book, 345 kV and above,* Chapter *12,* Second Edition, Revised, Palo Alto California, 1987.

[3] P. Chowdhuri, S. Li an P. Yan, "Rigorous analysis of back-flashover outages caused by direct lightning strokes to overhead power lines," Generation, Transmission and Distribution, IEE Proceedings, Vol. 149 Issue 1, Jan. 2002, pp.58-65

[4] M. E. Almeida and M. T. Correia de Barros, "Modeling of long ground electrodes for lightning studies," Eleventh International Symposium on High Voltage Engineering, (Conf. Publ. No. 467), Vol.2, 1999, pp: 267 –271.

[5] M. E. Almeida and M. T. Correia de Barros, "Accurate modeling of rod driven tower footing," *IEEE Trans. on Power Delivery*, Vol. 11, No. 3, July 1996. pp 1606-1609.

[6] EPRI, "Transmission Line Grounding" EPRI EL-2699, Vol. 1. Project 1494-1, October 1982.

[7] W. W. Lewis, "The Protection of Transmission Systems Against Lightning," Dover Publication Inc., New York, Chapter VII, 1.965, pp. 185-223.

[8] L. V. Bewley, "Traveling Waves on Transmission Systems," Dover Publication Inc., New York, Chapter 10, 1963, pp. 127-129/211-247

[9] Dawalibi, F., Baishiki, R., "Power frequency performance of transmission line structure grounds", IEEE transactions, Vol. PAS-103, No. 6, June 1984, pp1295-1303.

[10] E. R. Whitehead,"Protection of transmission lines," Lightning, Vol. 2, Edited by R.H. Golde, New York, Academic Press, 1977.

[11] Dawalibi, F., and Mukhedkar, D. "Parametric Analysis of Ground Systems", *IEEE Power Apparatus and Systems*, Vol. PAS-98, No. 6, pp. 2348-2354, 1979

[12] Dawalibi, F., Southey, R.D. and Ma, J. "Behavior of grounding systems in multilayer soils: A parametric analysis", *IEEE Transactions on Power Delivery*, Vol. 9, No. 1, pp. 334-342, 1994

[13] M. Bazaraa, J. Jarvis, H. Sherali, "Linear Programming and Network Flows," Wiley, 1990.

[14] Darveniza, M., et al, "Modeling for lightning performance calculations", IEEE Transactions on Power Apparatus and Systems, Vol. PAS-98, pp.1900-1908, 1979.

[15] International Electrotechnical Comission (IEC) Standard IEC 60-1, High-voltage test techniques Part 1: General definitions and test requirements, 1989.

[16] IEEE Working Group on Lightning Performance of Transmission Lines, "A simplified Method for Estimating Lightning Performance of Transmission Lines", IEEE Transactions on Power Apparatus and Systems, Vol. PAS-104, pp.919-932, April, 1985.

[17] Wiesner, C.J., "Hydrometheorology", Chapman and Hall Ltd., London, U.K., 1.970.

VIII. BIOGRAPHIES

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