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SMALL-SIZE UNIPLANAR COUPLED-FED PIFA FOR 2.4/5.2/5.8 GHz WLAN OPERATION IN THE LAPTOP COMPUTER

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ABSTRACT: *A coupled-fed PIFA (printed inverted-F antenna) printed on a small-size FR4 substrate promising for 2.4/5.2/5.8 GHz WLAN (wireless local area network) operation in the laptop computer is presented. When mounted at the top edge of the system ground plane or supporting metal frame of the laptop display, the PIFA shows a height of 9 mm and a small width of 7 mm only. With a narrow width, it is promising for more internal antennas to be mounted along the top edge of the supporting metal frame of the laptop display. The PIFA also shows a uniplanar structure, allowing it easy to fabricate, and is fed using a T-shaped coupling strip, which leads to successful excitation of two wide operating bands to cover WLAN operation in the 2.4 GHz band and 5.2/5.8 GHz band. Details of the proposed PIFA are studied, and experimental and simulation results are presented.* © 2009 Wiley Periodicals, Inc. Microwave Opt Technol Lett 51: 1023–1028, 2009; Published online in Wiley InterScience (www.interscience.wiley.com). DOI 10.1002/mop.24206

Key words: *internal laptop computer antenna; PIFA; WLAN antenna; dual-band antenna; coupled-fed PIFA*

1. INTRODUCTION

Internal WLAN (wireless local area network) antennas have become a standard embedded element in general laptop computers for wireless internet access. A variety of promising dual-band or triple-band WLAN antennas to be embedded inside the laptop computer for 2.4/5.2/5.8 GHz band operation have also been reported in the literature [1–12]. These internal WLAN antennas include the slot antenna [2], the traditional metal-plate or printed PIFA (planar inverted-F antenna) with a direct feed [3–6], the monopole antenna [7–11], the coupled-fed PIFA [12], and so on. These reported WLAN antennas [2–11], however, generally show a width of about 20 mm or larger along the top edge of the system ground plane or the supporting metal frame of the laptop display, when embedded inside the laptop computers. For the coupled-fed PIFA reported in [12], the required width of the antenna is reduced to be 13 mm only; however, the obtained impedance bandwidths at about 2.4 and 5.5 GHz cannot cover WLAN operation in the 2.4 GHz band (2400–2484 MHz for IEEE 802.11 b/g) and 5.2/5.8 GHz bands (5150–5350/5725–5875 MHz for IEEE 802.11a).

Recently, owing to the increasing numbers of the internal antennas to be embedded inside the laptop computer for practical applications such as the internal WWAN (wireless wide area network) antennas for GSM/DCS operation [13–15], three or more internal WLAN antennas for MIMO (multiple input multiple output) operation [16, 17] and the like, the width of the internal antennas for laptop computer applications is required to be as small as possible. In addition, for promising applications in the

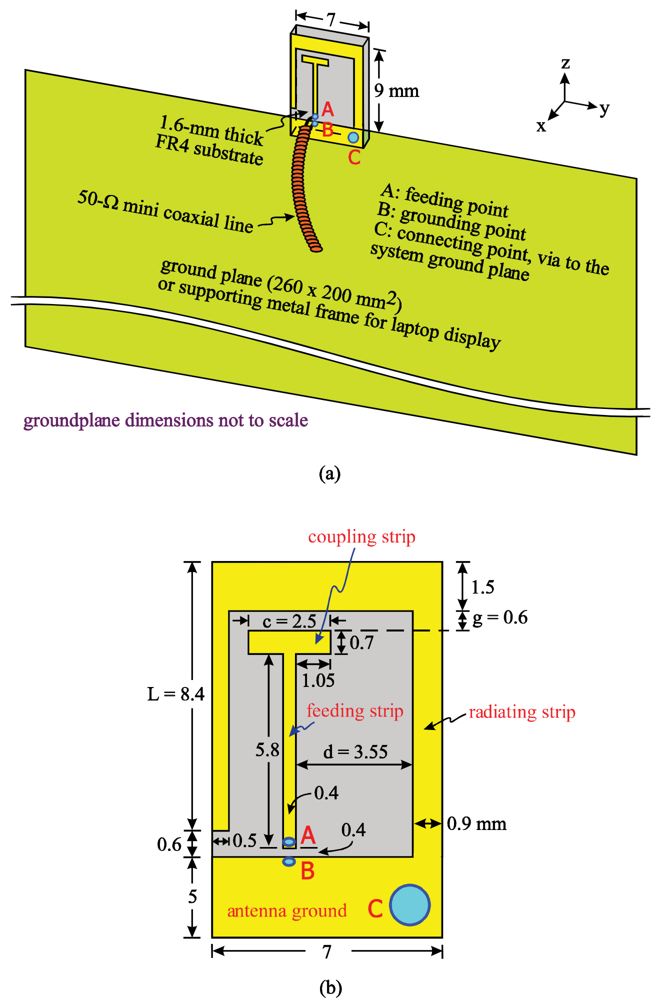

Figure 1 (a) Geometry of the uniplanar coupled-fed printed PIFA for 2.4/5.2/5.8 GHz WLAN operation in the laptop computer. (b) Detailed dimensions of the metal pattern of the printed PIFA. [Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com]

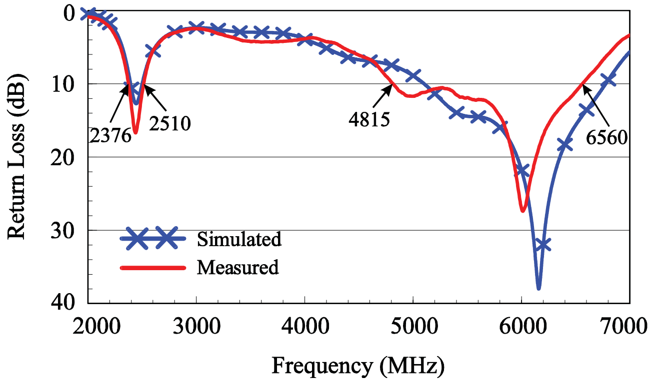

Figure 2 Measured and simulated return loss of the proposed PIFA. [Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com]

thin-profile laptop computers, which receive much attention recently, planar internal antennas with a thin thickness are demanded for such applications. To meet the challenging requirements of small width and thin thickness, we present in the article a novel uniplanar printed PIFA with a height of 9 mm and a small width of 7 mm only, when the antenna is mounted at the top edge of the system ground plane of the laptop computer. The antenna height of 9 mm is less than that (about 10 mm) of the internal WWAN antennas [13–15] and is promising for practical applications. For the narrow width, it is much smaller than those reported in the literature [1–12]; this allows for more internal antennas to be mounted along the top edge of the supporting metal frame of the laptop display. With the uniplanar structure and small size, two wide operating bands for covering the 2.4 and 5.2/5.8 GHz bands for WLAN operation are also obtained. Details of the proposed small-size PIFA for application as an internal WLAN antenna in the laptop computer are described. Results of the fabricated prototype of the proposed PIFA are also presented and discussed.

2. PROPOSED SM ALL-SIZE UNIPLANAR COUPLED-FED PIFA

Figure 1(a) shows the geometry of the proposed PIFA for 2.4/5.2/5.8 GHz WLAN operation in the laptop computer. The PIFA has a simple uniplanar structure and is printed on one side of the 1.6-mm thick FR4 substrate of relative permittivity 4.4. The PIFA is mounted at the center of the top edge of the system ground plane of length 260 mm and width 200 mm, which are reasonable dimensions for the supporting metal frame of the display of the general laptop computer. Detailed dimensions of the metal pattern of the uniplanar printed PIFA are shown in Figure 1(b).

The proposed PIFA comprises a folded radiating strip, a T-shaped feeding strip, and an antenna ground. The folded radiating strip with its one end short-circuited to the antenna ground of size 5×7 mm^2 is coupled-fed by the T-shaped feeding strip. When the proposed PIFA is mounted at the top edge of the system ground plane, the antenna ground is electrically connected to the system ground plane at point C (the connecting point) through the FR4 substrate as shown in the figure. The coupling feed [18, 19] used in the proposed PIFA can result in successful excitation of two wide operating bands at about 2.4 and 5.5 GHz for the desired WLAN operation. When the traditional direct contact feed is used, the radiating strip of length about 24 mm (about 0.2 wave length at 2.4 GHz) can generate a quarter-wavelength resonant mode at about 2.4 GHz only; the desired resonant mode at about 5.5 GHz for WLAN operation in the 5.2/5.8 GHz bands cannot be achieved. Detailed results are presented in Figure 3; with the aid of Figure 3, the comparison of the proposed PIFA and the corresponding traditional PIFA with a direct contact feed will be discussed in Section 3.

The T-shaped feeding strip is located at a distance d of 3.55 mm to the short-circuited end of the folded radiating strip. The location d can effectively control the central frequencies of the two desired operating bands, especially the lower one for the 2.4 GHz WLAN operation. Another effective way of controlling the lower resonant mode is to adjust the length L of the open-end section of the folded radiating strip. Through adjusting the length L, the total resonant length of the folded radiating strip is varied, hence, resulting in the variation of the excited resonant mode contributed by the radiating strip. Detailed effects of the length L and the location d on the antenna performance are analyzed with the aid of Figures 4 and 5 in Section 3.

The T-shaped feeding strip is further formed by a vertical strip and a horizontal strip. The vertical strip has a length of 5.8 mm and a width of 0.4 mm, while the horizontal strip functioned as the coupling strip has a length c of 2.5 mm and a width of 0.7 mm. Through the coupling gap (width g = 0.6 mm), the feeding strip capacitively excites the radiating strip. Because the contributed

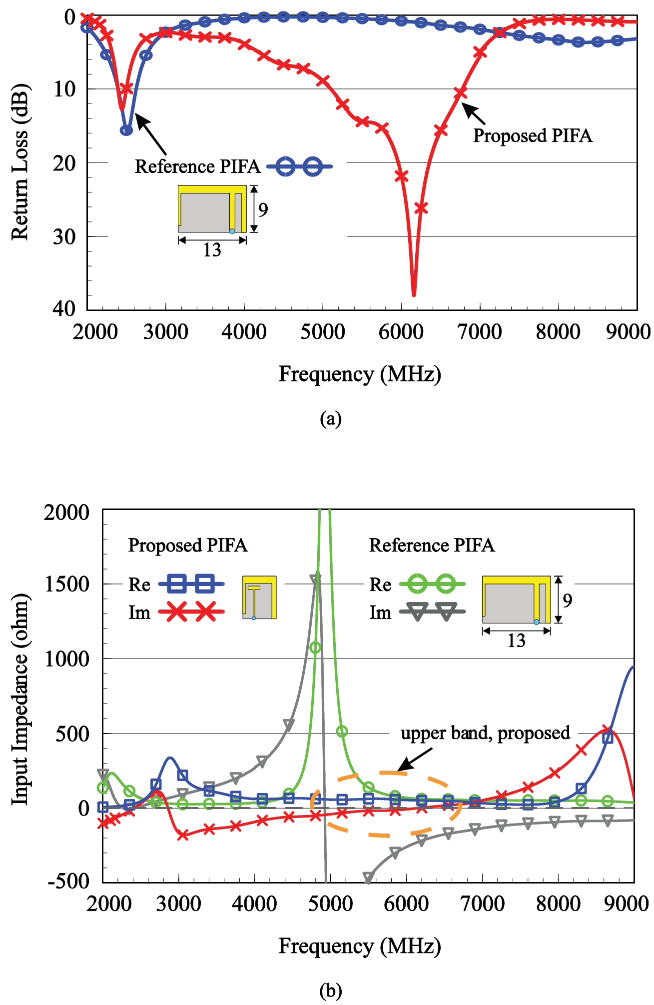

Figure 3 Comparison of (a) the simulated return loss and (b) the simulated input impedance of the proposed PIFA and the reference PIFA (the traditional PIFA with a direct contact feed for 2.4 GHz band operation). [Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com]

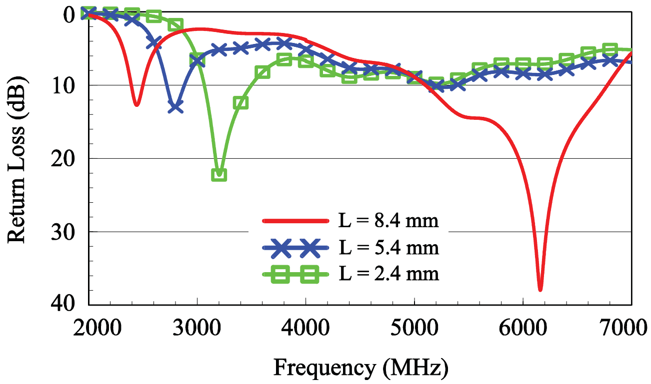

Figure 4 Simulated return loss as a function of (a) the length L of the open-end section of the folded radiating strip. Other parameters are the same as given in Figure 1. [Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com]

capacitance of the proposed coupling feed can be easily controlled by adjusting the coupling-strip length c and the coupling-gap width g, the impedance matching of the two desired operating bands at about 2.4 and 5.5 GHz can be adjusted by tuning these two parameters. More detailed results of the parameters c and g on the performances of the proposed PIFA will be discussed with the aid of Figure 6 in the next section. Also note that across the small gap of 0.4 mm between point A (the open-end of the feeding strip) and point B (the grounding point at the antenna ground), a 50-Ω mini coaxial line is applied in this study to test the proposed PIFA, and measured results are presented in Section 3 for discussion.

3. RESULTS AND DISCUSSION

Figure 2 shows the measured and simulated return loss of the fabricated prototype. Agreement between the measured data and the simulated results obtained using Ansoft high frequency structure simulator (HFSS) [20] is obtained. Two operating bands at about 2.4 and 5.5 GHz are excited with good impedance matching. For the lower band at about 2.4 GHz, the measured 10-dB return-loss impedance bandwidth reaches 134 MHz (2376–2510 MHz), which covers WLAN operation in the 2.4 GHz band. For the upper band at about 5.5 GHz, the antenna shows a wide bandwidth of 1745 MHz (4815–6560 MHz) to easily cover WLAN operation in the 5.2/5.8 GHz bands.

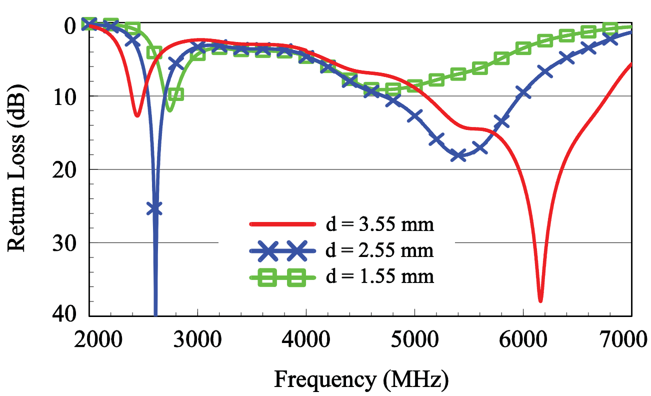

Figure 5 Simulated return loss as a function of the location d of the feeding strip of the proposed PIFA. Other parameters are the same as given in Figure 1. [Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com]

Figure 3 shows the comparison of the simulated return loss and input impedance of the proposed PIFA and the reference PIFA (the traditional PIFA with a direct contact feed for 2.4 GHz band operation). From the simulated return loss shown in Figure 3(a), there is only one resonant mode excited at about 2.4 GHz for the reference PIFA. For the proposed PIFA, owing to the use of the coupling feed, a wide operating band at about 5.5 GHz is generated. This behavior can be seen more clearly from the simulated input impedance in Figure 3(b). The input impedance level at around 5 GHz for the reference PIFA shows a very large value, which makes it difficult to generate the desired upper band for WLAN operation. On the other hand, by using the coupling feed, the proposed PIFA shows a fairly smooth input impedance level in the frequency range of about 5 to 7 GHz, leading to successful excitation of the desired upper band as seen in Figure 3(a) for 5.2/5.8 GHz WLAN operation.

Figure 4 shows the simulated return loss as a function of the length L of the open-end section of the folded radiating strip; other parameters are the same as given in Figure 1. With a decrease in the length L, the lower band is shifted to higher frequencies as expected. However, the successful excitation of the upper band is achieved for the case of $L = 8.4$ mm only. This behavior suggests that the proper coupling of the open-end section with the antenna ground is also important in the proposed design. When the length L is larger, the gap between the open-end section and the antenna ground decreases, which results in increased coupling. This behavior helps improve the impedance matching as the coupling feed

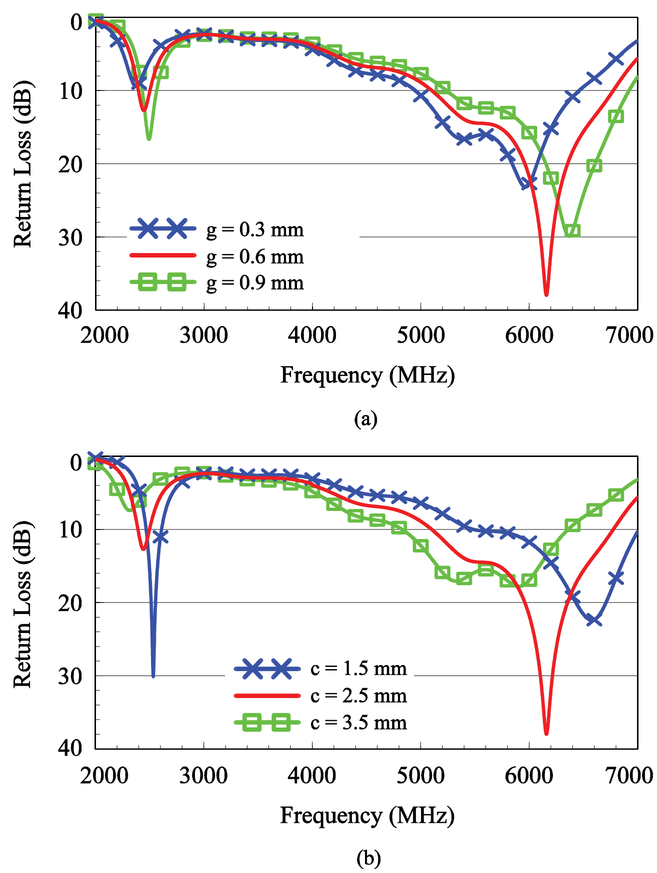

Figure 6 Simulated return loss as a function of (a) the coupling-gap width g and (b) the coupling-strip length c of the proposed PIFA. Other parameters are the same as given in Figure 1. [Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com]

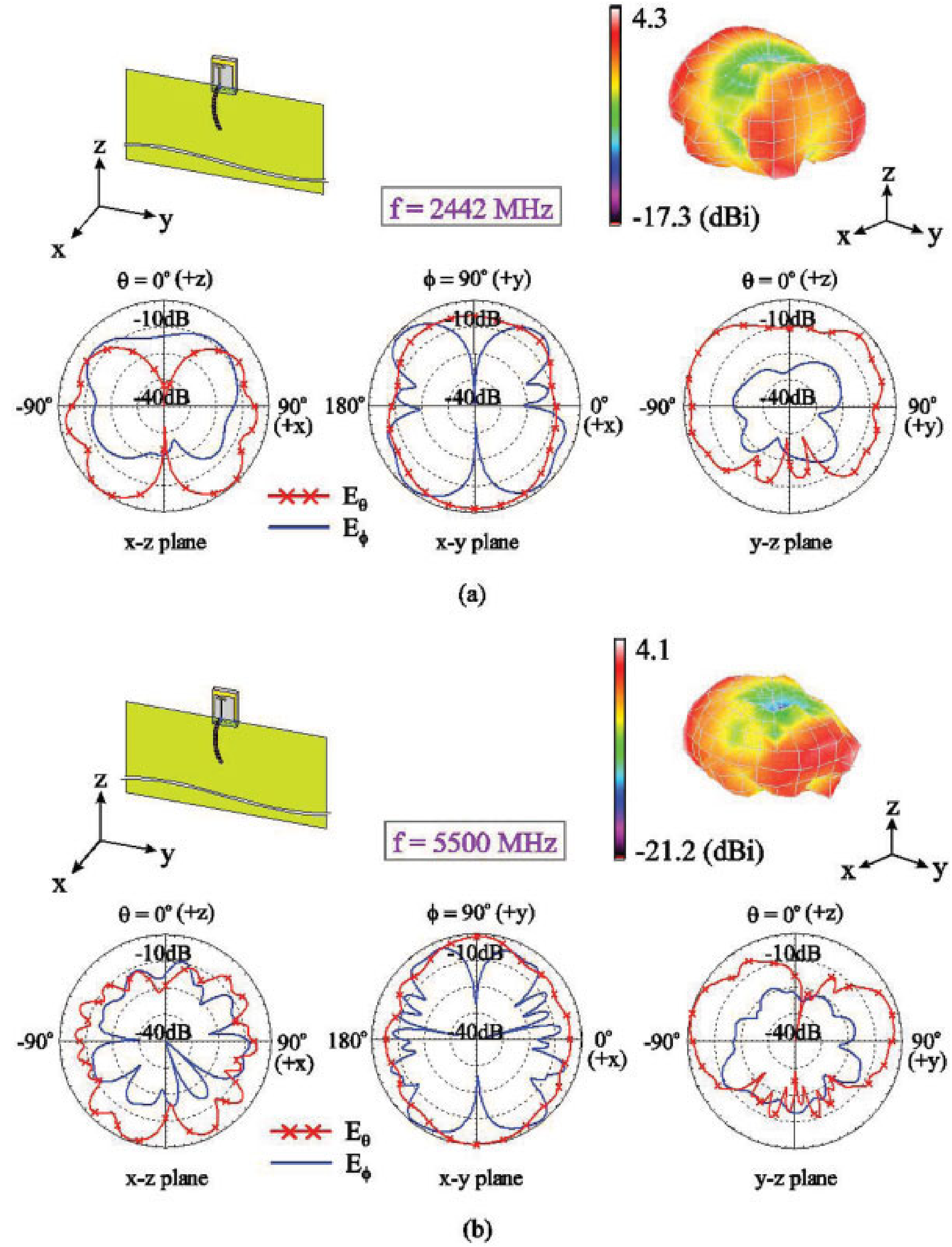

Figure 7 Measured 3D and 2D radiation patterns at (a) 2442 MHz and (b) 5500 MHz for the proposed PIFA. [Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com]

does for the proposed PIFA, resulting in successful excitation of the desired upper band for the proposed PIFA.

Effects of the location *d* of the feeding strip are studied in Figure 5. Results of the simulated return loss for the location *d*, which varied from 1.55 to 3.55 mm are presented. From the results obtained, the two desired lower and upper bands can be fine-adjusted to cover 2.4 and 5.5/5.8 GHz WLAN operation by properly selecting the location *d*. In this study, the preferred location *d* is selected to be 3.55 mm.

Figure 6 presents the simulated return loss as a function of the coupling-gap width *g* and the coupling-strip length *c* of the proposed PIFA. The results for the coupling-gap width *g* varied from 0.3 to 0.9 mm are shown in Figure 6(a); other parameters are the same as given in Figure 1. When the width *g* is varied, it is expected that the contributed capacitance of the coupling feed to the antenna's input impedance is varied. This behavior leads to the impedance matching variations seen in the figure. For the results of the coupling-strip length *c* shown in Figure 6(b), similar behavior is seen. These results indicate that adjusting the coupling-gap width *g* and the coupling-strip length *c* is an effective way in achieving good impedance matching for frequencies over the two desired lower and upper bands at about 2.4 and 5.5 GHz.

Radiation characteristics of the proposed PIFA are also studied. Figure 7 plots the measured three-dimensional (3D) and two-dimensional (2D) radiation patterns at 2442 and 5500 MHz, central frequencies of the antenna's lower and upper bands. The 3D patterns show the antenna's total radiation power, whereas the 2D patterns plot the E_θ and E_ϕ components separately. As seen from the 2D pattern in the azimuthal plane (*x-y* plane), comparable E_θ and E_ϕ components are observed; this is advantageous for practical applications, especially for indoor WLAN operation where the wave propagation is usually complex. Figure 8 shows the measured antenna gain and radiation efficiency. For frequencies over the lower band shown in Figure 8(a), the antenna gain is in the range of about 3.0–4.3 dBi, and the radiation efficiency is better than 67%. Figure 8(b) shows the results for the upper band. The antenna gain is varied from about 4.0 to 5.5 dBi and the radiation efficiency is also better than 67%.

It is also noted that the average antenna gain, defined as the average of the antenna gain over all of the ϕ angles in the azimuthal plane (*x-y* plane), is an important factor for practical applications of the internal WLAN antenna in the laptop computers. The internal antenna to be applied should meet the minimum average antenna gain requirement as shown in Table 1 (the specification shown in the table) for practical applications (Yageo Corporation, Private Communication, http://www.yageo.com.tw). The measured results of the average antenna gain for the proposed PIFA are presented in Table 1. Results indicate that the average antenna gain of the proposed PIFA is much better than that required for practical applications, even when including the power loss of the long mini coaxial line (generally about 70 mm) connected to the internal antenna in the laptop computer, which is estimated to be about −2 dB for frequencies in the 2.4 GHz band and about −4 dB in the 5.2/5.8 GHz bands (Yageo Corporation, Private Communication, http://www.yageo.com.tw).

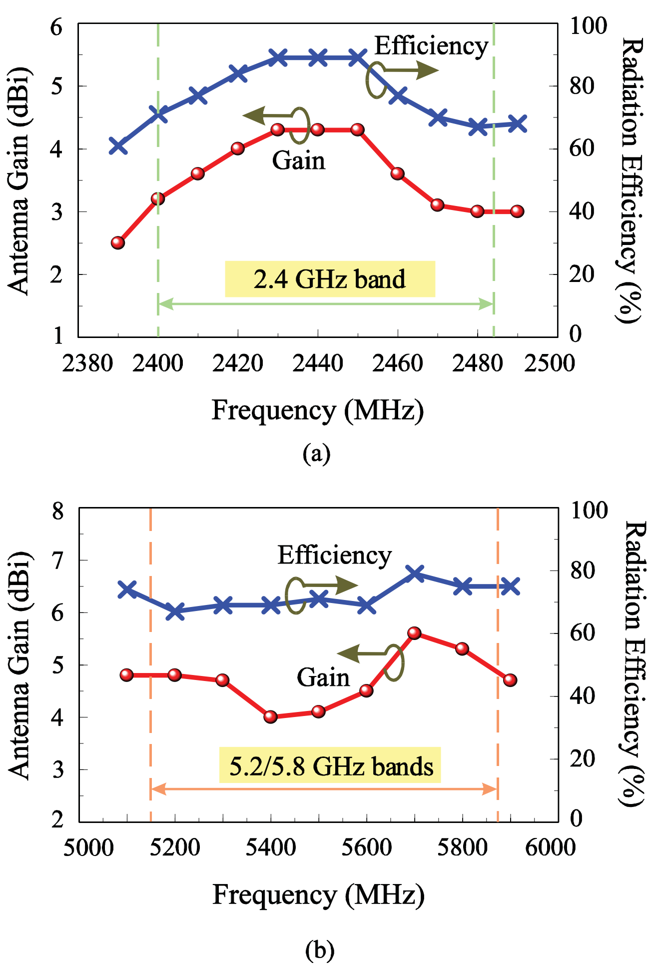

Figure 8 Measured antenna gain and radiation efficiency for the proposed PIFA. (a) Lower band. (b) Upper band. [Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com]

TABLE 1 Measured Average Antenna Gain in the Azimuthal Plane for the Proposed PIFA

Average antenna gain (dBi)		Proposed PIFA (dBi)	Specification (dBi)
2.4 GHz band (802.11b/g)	2400 MHz	−0.1	−4.0
	2442 MHz	−0.2	−4.0
	2484 MHz	−0.8	−4.0
5.2/5.8 GHz bands (802.11a)	5150 MHz	1.4	−5.0
	5350 MHz	1.2	−5.0
	5500 MHz	1.2	−5.0
	5725 MHz	1.6	−5.0
	5875 MHz	1.2	−5.0

The specification is the minimum average antenna gain required for practical applications of the internal WLAN antenna in the laptop computers (Yageo Corporation, Private Communication, http://www.yageo.com.tw).

4. CONCLUSIONS

A small-size uniplanar printed PIFA with a coupling feed capable of providing two wide operating bands for covering 2.4/5.5/5.8 GHz WLAN operation in the laptop computer has been proposed and studied. The proposed PIFA is easily printed on one side of the inexpensive FR4 substrate at low cost, and shows a height of 9 mm and a small width of 7 mm only above the top edge of the system ground plane or supporting metal frame of the laptop display. In addition to the wideband operation and small size achieved, the proposed PIFA shows good radiation characteristics for frequencies over the desired lower and upper bands. The obtained average antenna gain in the azimuthal plane of the proposed PIFA also meets the required specification for practical applications of the internal WLAN antenna in the laptop computers.

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AN UNCONDITIONALLY STABLE WAVE EQUATION PML ALGORITHM FOR TRUNCATING FDTD SIMULATION

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ABSTRACT: *An unconditionally stable (US) wave equation (WE) perfectly matched layer (PML) absorbing boundary condition is implemented for two-dimensional (2-D) open region finite-difference time-domain (FDTD) simulation by virtue of weighted Laguerre polynomial expansion. This novel PML preserves unconditional stability as well as comparative accuracy to the original wave equation PML (WEPML). Numerical examples are included to verify high accuracy and efficiency of the proposed algorithm.* © 2009 Wiley Periodicals, Inc. Microwave Opt Technol Lett 51: 1028–1032, 2009; Published online in Wiley InterScience (www.interscience.wiley.com). DOI 10.1002/mop.24233

Key words: *perfectly matched layer; Laguerre polynomials; finite-difference time-domain; wave equation; unconditionally stable*

1. INTRODUCTION

The finite-difference time-domain (FDTD) method has been widely utilized to solve Maxwell's equations in numerous electromagnetic applications since it was originally proposed [1]. Later, the scalar wave equation (WE) FDTD method was proposed and proved to be a memory-efficient method [2]. In the WE-FDTD method, the field components are decoupled, and only one field component needs to be computed if the values of other field components are not required. Hence, less memory requirements are needed in comparison with the conventional FDTD method. A reduction of 33% on memory requirements for a two-dimensional (2-D) problem was reported in [2]. When this WE- FDTD is used for solving open region problems, accurate absorbing boundary conditions (ABCs) are needed for truncating the computational domain. Recently, the scalar wave equation PML (WEPML) algorithm [3] was introduced to truncate the WE-FDTD domains. The unconditionally stable (US) WEPML formulations based on the Crank- Nicolson (CN) FDTD, alternating direction implicit (ADI) FDTD, and locally one- dimensional (LOD) FDTD schemes have been introduced in [4–6].

Although the Courant-Friedrich-Levy (CFL) stability condition limit is removed in the aforementioned unconditionally stable WEPML methods, the time step is still bounded by the accuracy requirement [6], because large numerical error is introduced if large time step is used. The absorbing effects of the existent unconditionally stable WEPMLs, including CN-WEPML, ADI-WEPML, and LOD-WEPML, become worse as the time step increases [4–6]. In this letter, the unconditionally stable scheme based on weighted Laguerre polynomials [7] is introduced into the construction of a novel unconditionally stable WEPML algorithm. This new unconditionally stable WEPML algorithm is proved to be more efficient than the original WEPML yet maintain high accuracy. Numerical examples are included to demonstrate the performance of this new method.

2. FORMULATION

Consider a source free, homogenous, lossy, and nondispersive computational domain. With the stretched coordinate PML (SC-PML) formulation [8], Maxwell's equations in the PML regions can be written as

$$\nabla_s \times \mathbf{E} = -j\omega\mu_0\mathbf{H} \tag{1}$$

$$\nabla_s \times \mathbf{H} = j\omega\varepsilon_0\left(\varepsilon_r + \frac{\sigma}{j\omega\varepsilon_0}\right)\mathbf{E} \tag{2}$$

where μ_0 and ε_0 are the permeability and permittivity in free space, respectively; ε_r and σ are, respectively, the relative permittivity and the conductivity of the medium; and

$$\nabla_s = \sum_{\eta=x,y,z} \hat{a}_\eta \frac{1}{S_\eta}\frac{\partial}{\partial\eta},\ S_\eta = 1 + \frac{\sigma_\eta}{j\omega\varepsilon_0} \tag{3}$$

where σ_η is the conductivity profile in the PML regions along the η direction. Because the PML regions are source free, that is, $\nabla \cdot \mathbf{E} = 0$, the scalar wave equation in terms of E_z field component in a 2-D TM_z problem can be obtained as [3]

$$\frac{(j\omega)^2}{c^2}\left(\varepsilon_r + \frac{\sigma}{j\omega\varepsilon_0}\right)E_z = \sum_{\eta=x,y}\frac{1}{S_\eta}\frac{\partial}{\partial\eta}\frac{1}{S_\eta}\frac{\partial}{\partial\eta}E_z \tag{4}$$

where $c = 1/\sqrt{\mu_0\varepsilon_0}$ is the speed of light in free space. Eq. (4) can also be written as

$$\left(j\omega\varepsilon_r + \frac{\sigma}{\varepsilon_0}\right)E_z = j\omega\sum_{\eta=x,y}\frac{c}{j\omega + \sigma_\eta/\varepsilon_0}\frac{\partial}{\partial\eta}\frac{c}{j\omega + \sigma_\eta/\varepsilon_0}\frac{\partial}{\partial\eta}E_z \tag{5}$$

By introducing auxiliary variables