Abstract—The fault detection and analysis for power transformer are the key measures to improve the security of power systems and the reliability of power supply. Due to the complicity of the power transformer structure and the variations in operating conditions, the occurrence of a fault inside power transformer is uncertain and random. Until now, the fault statistics of power transformer is very limited due to the low fault rate. A novel fault tree analysis method based on fuzzy set theory is proposed for power transformer. Using this method, the index of fault rate can be converted into fuzzy number of fault rate. The method of expert grading can be used to perform the probability of fault estimation without the requirement for corresponding statistics information. The details of fuzzy number design are described in the paper and an application example of the method is also provided. The results show that the proposed fuzzy fault tree analysis method is flexible and adaptive for fault diagnose of power transformer. Therefore, it is a useful engineering tool for the fault analysis and prevention of power transformer.

Index Terms—Power transformer; Fault analysis; Fault tree; Fuzzy sets; System reliability.

I. INTRODUCTION

Power transformers (power transformers) are important equipments of power systems. Therefore, the fault diagnosis and analysis of power transformer are important issues for power systems. One of effective diagnosis methods suitable for power transformer is the fault tree analysis. The conventional fault tree analysis is based on probability theory. Usually, the failure probabilities can only be estimated based on a large amount of measured data. However, it is unlikely that enough real data can be collected for the purpose of accurate estimation. Moreover, the fault on power transformers caused by the mal-operation of operating staff occurs randomly, which makes the application of the conventional fault tree analysis difficult.

Fuzzy-sets theory can be the best tool to solve the above mentioned problem, which is able to analysis the fault events without the need for exact information of fault probability.

The fault tree analysis method can not only find the degree of the severity of a local fault and find its weakness, but also determine the quantification risk index of power systems.

Both probability theory and accurate fault rate are the basis for the fault tree analysis method. To achieve good performance, it needs a large amount of data to analyze the behavior of the transformer. However, the fault rate of power transformer is very low and the fault behavior is very complicated, which will result in the difficult in collecting enough data for analysis. The uncertainty of operation factors, i.e., mal-operation by staff, environment and climate change, the random nature of the fault and so on, will make it impracticable to accurately collect and record the fault for the power transformers.

The fuzzy theory is proposed to analyze the conventional fault tree by applying triangle fuzzy probability theory (TFPT) instead of the accurate probability theory. The TFPT can locate the analysis results satisfying the engineering requirements based on the fuzzy operation for the fault rate statistics. The fuzzy probability value can be acquired based on the fuzzy operation for the basic behavior without statistical information. There exist different expression modes (i.e., language value, accurate value, section value and so on) for the basic behavior with statistical information. Such different data information must be converted into triangle fuzzy number [2][3], as shown in Fig.1.

The analysis method can be used to process the probability of the fault without the statistical data. The methods for the different mode probability data conversion to triangle fuzzy number are introduced in the paper. The experimental results based on the fault tree of the power transformer prove the effectiveness of the proposed method.

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II. EXPRESSION AND OPERATOR OF FUZZY NUMBER

A. Expression of the triangle fuzzy number

The expression of the triangle fuzzy number can be illustrated as follows:

\[ \tilde{A} = \langle l, m, u \rangle \]

The expression above is defined in reference [2]. The subjection function is expressed in Fig.1. And the subjection function for the triangle fuzzy number is formed by linear function shown as follows:

\[ \mu_A(x) = \begin{cases} 
\frac{(x-m+l)}{l} & l \leq x \leq m \\
\frac{(m+u-x)}{u} & m \leq x \leq u \\
0 & \text{others}
\end{cases} \]  (1)

where, \( l \) and \( u \) are the lower and upper limit of the undetermined data, \( m \) is barycenter value.

B. Algorithm of the triangle fuzzy number

The arithmetic operation results based on the constant value, i.e., plus, subtract and multiply, are still the triangle fuzzy number. Usually, such operations, i.e., multiply, divide, and reciprocal results are not in the triangle fuzzy number form. However, they can be approximated by the triangle fuzzy number. The algorithm operations for the triangle fuzzy number, illustrated as \( \langle l, m, u \rangle \), and the formula of the fuzzy AND gate and fuzzy OR gate are given in reference [4].

III. THE TRIANGLE FUZZY EXPRESSION FOR BASIC EVENTS

There are two typical methods to analyze the fault tree for the basic events: the event with statistical data and the event without statistical data. Three types of probability value form the event with and without statistical data are: accurate, language and non-triangle fuzzy.

A. Conversion of the accurate probability value

The accurate probability value can not express the engineering practice situation or the fault behavior induced by the uncertainty factors. Therefore the fault tree should be converted to the fuzzy mode data for the convenience of the fuzzy operations. An example is used to illustrate the conversion method. \( p \) is a guessed accurate probability value. And the triangle fuzzy number formation for \( p \) is expressed as \( \tilde{q} = \langle l, m, u \rangle \), where \( p \) is considered as the barycenter value, \( m \) is for the triangle fuzzy number formation. The system composed of both experts and technicians will analyze and estimate the uncertainty. The estimation will take influence of people, the randomness of the events, and the weather condition into account. The expert system will produce an undetermined datum or estimated datum for fault rate based on practical situations. The lower and upper limit of the undetermined data can be expressed \( l \) and \( u \) respectively. For example, lower limit and upper limit of a certain probability value, \( p \) are given by 0.1 and 0.2 respectively. And the accurate probability value, \( p \), can be converted to the triangle fuzzy number formation, as shown \( \langle 0.1, p, 0.2 \rangle \).

B. Conversion of the non-triangle fuzzy number

In the probability expressions, there is non-triangle fuzzy number form, i.e., trapeziform distribution and normal distribution, which express fuzzy number belonging to trapeziform data and normal data. It is important to convert the non-triangle fuzzy number form to triangle fuzzy number form. Firstly, we should find the barycenter value of the subjection function \( m \). Secondly, the barycenter value, \( m \), must be converted to triangle fuzzy number form, as shown in \( \langle l, m, u \rangle \).

C. \( 3\sigma \) expression mode

It is difficult to collect statistical data for the location of fault behavior of the power transformers. Therefore, it is necessary to find other ways to collect the key data from the fault location in power transformers. The \( 3\sigma \) expression method is proposed to find the probability value of the fault location in power transformers. The principle of the \( 3\sigma \) expression method can be generalized as follows: the fuzzy probability value of the fault event can be determined by the expert assess method. The assess system is composed of 3 experts. Each expert will give the estimate value for the probability value of the fault event, each average value of the probability value of the fault event is the same and given by \( m \), their difference are the same and shown as \( \sigma \). The guessed probability values belong to normal distribution. According to the \( 3\sigma \) law, the factual value is located in the section of \( \left[ m-3\sigma, m+3\sigma \right] \) with probability value of 99.7%. And guess \( l = u = 3\sigma \), each probability value of the fault event can be expressed as \( \langle 3\sigma, m, 3\sigma \rangle \).

IV. ANALYSIS OF THE FUZZY IMPORTANCE DEGREE

Analysis of the fuzzy importance degree is the important issue of the fault tree analysis method. According to the definition of the median of the fuzzy number, the median analysis method of the fuzzy importance degree can be expressed as follows:

In the Fig.1, let \( A_1 = \int_{m-1}^{m} u_A(x)dx \), \( A_2 = \int_{m}^{m+u} u_A(x)dx \), and \( A = A_1 + A_2 \), there exists a point, \( Z \), which can pass the division, the right area under the fuzzy curve is equal to that of the left. And the point, \( Z \), is defined as the median of the fuzzy number.

For the fault tree of the structure function, shown as \( \Phi(x_1, x_2, \cdots, x_n) \), the median of the fuzzy number can be marked as \( T_z^l \). The median of the fuzzy number of the basic event is marked as \( T_z^l \).
The importance degree of the basic event, shown as $x_i$, is expressed as follows:

$$S_i = T_z - T_z^I$$  \hspace{1cm} (2)$$

If $S_i > S_j$, the importance degree for $x_i$ is higher than that of $x_j$. According to the importance degree, the order of every fuzzy number of the basic event can be determined.

V. FAULT TREE ANALYSIS BASED ON FUZZY THEORY

The fault tree analysis based on fuzzy theory can be achieved as the following eight steps.

Step 1: selection of proper event, confirmation of the fault tree using every kinds of logic gate, and conversion of non-and-or gate to non-and-or gate.

Step 2: arrangement of the fault tree according to statistical property of the sampled data, generalization of two kinds of fault trees: fault tree with statistical data and fault tree without statistical data.

Step 3: experiencing data and other useful ways to acquire fault rate of the fault tree according to reliability manuals; forming the probability distribution parameters and other parameters according to fault rate. Then, an accurate probability can be determined.

Step 4: The fuzzy probability can be acquired for the fault tree without statistical data adopting all kinds of fuzzy number and language evaluation based on the decision by the experts.

Step 5: According to a ruler, both of the non-triangle fuzzy and language value from step 3 and step 4 must be converted to uniform triangle fuzzy number.

Step 6: In order to acquire the least secant set of the fault tree, probability value of the event can be acquired according to and-or gate fuzzy operator.

Step 7: analysis of the importance degree of the basic event.

Step 8: analysis the result of the fuzzy fault and bring forward analysis advice.

VI. ANALYSIS EXAMPLE

The fault tree of the bushing of the power transformers is taken as the analysis example to explain the analysis process for the fuzzy fault tree.

Step 1: The fault tree of the power transformers can be illustrated as in Fig.2. For simplifying the analysis, the analysis of the fault tree of the bushing of the power transformers can be an analytical example. The fault tree of the bushing of the power transformers is shown in Fig.3.

Step 2: Sort the fault event into two kinds: fault tree with statistical data and fault tree without statistical data. In Fig.3, E4, E8, E10, E11 and E12 are fault trees with statistical data, the others are fault trees without statistical data.

Step 3: According to the data from the reference [1], the accurate probability value of the fault tree with statistical data can be acquired as: $[E4=0.10, E8=0.11, E11=0.08, E11=0.20, \text{and } E12=0.07]$.

Step 4: according to scores from the experts and $3\sigma$ expression ruler, the probability value of the fault tree without statistical data can be estimated. The estimated results are illustrated in Table 1.

Step 5: all of the probability value of the fault tree must be converted to uniform triangle fuzzy number. The conversion method for the accurate probability value is illustrated in part A of section III; scores from the experts can be converted due to $3\sigma$ expression ruler.

The fluctuation range for the fault tree with statistical data is shown as follows: $E4(-0.05, +0.10), E8(\pm 0.08), E10(-0.07, +0.10), E11(-0.11, +0.09), E12(-0.01, +0.05)$. Such estimated results can be converted to the uniform triangle fuzzy number, as shown in Table 2.

Step 6: to acquire the least secant set of the fault tree of the bushing of the power transformers, acquire probability value of the event according to and-or gate fuzzy operator. And probability value of the bushing of the power transformers can be determined accordingly.

The acquiring method of the least secant set of the bushing can be expressed as follows:

All the least secant set of the bushing can be shown as: $\{K1=E1, K2=E2, K3=E3, K4=E4, K5=E5, K6=E6, K7=E7, K8=E8, K9=E9, K10=E10, K11=E11, K12=E12, K13=E13, K14=E14, K15=E15, K16=E16, K17=E17, K18=E18\}$.

The probability value of the bushing of the power transformers can be determined:

$$\hat{Q} = 0.102, 0.173, 0.091$$

The changing range of the probability value of the bushing of the power transformers is shown as $0.071-0.264$.

Step 7: Analysis of the importance degree of the basic event. The median values of the fault event are shown $\{E4=0.134, E10=0.084, E11=0.20\}$.

Step 8: the importance degree of the fault event can be assorted as:

$\{E18=E16>E11>E7>E17>E6>E4>E15>E3>E8>E5>E9>E1>E10>E2>E12>E13=E14\}$.

An important conclusion can be drawn that operation mistake made by people, damp lighting/dirt lighting fault, and improper structure fault are of the greatest influence. The personnel cultivation and timely remove dirt from the bushing of the power transformers will greatly reduce the occurrences of the power transformer faults and lengthen its life span.

VII. CONCLUSIONS

Fuzzy-sets theory is useful for studying the reliability of power transformers. Combined with the fault tree analysis, it is able to analyze the fault efficiently, without the disadvantages of conventional fault tree analysis method. The engineering analysis of a practical application verified the effectiveness of the proposed method. The fuzzy fault tree method can be easily designed with an aim of prevention of the fault.
Fig. 2 Fault tree of the power transformer

Fig. 3 Fault tree of the bushing of power transformer

TABLE I SCORES FROM THE EXPERTS

<table>
<thead>
<tr>
<th>Code</th>
<th>Expert_1</th>
<th>Expert_2</th>
<th>Expert_3</th>
<th>m</th>
<th>σ</th>
<th>Code</th>
<th>Expert_1</th>
<th>Expert_2</th>
<th>Expert_3</th>
<th>m</th>
<th>σ</th>
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</thead>
<tbody>
<tr>
<td>E1</td>
<td>0.08</td>
<td>0.09</td>
<td>0.10</td>
<td>0.09</td>
<td>0.00</td>
<td>E9</td>
<td>0.09</td>
<td>0.10</td>
<td>0.09</td>
<td>0.09</td>
<td>0.00</td>
</tr>
<tr>
<td>E2</td>
<td>0.08</td>
<td>0.075</td>
<td>0.09</td>
<td>0.08</td>
<td>0.00</td>
<td>E13</td>
<td>0.05</td>
<td>0.06</td>
<td>0.08</td>
<td>0.06</td>
<td>0.01</td>
</tr>
<tr>
<td>E3</td>
<td>0.10</td>
<td>0.15</td>
<td>0.12</td>
<td>0.12</td>
<td>0.02</td>
<td>E14</td>
<td>0.05</td>
<td>0.06</td>
<td>0.08</td>
<td>0.06</td>
<td>0.01</td>
</tr>
<tr>
<td>E5</td>
<td>0.10</td>
<td>0.10</td>
<td>0.09</td>
<td>0.09</td>
<td>0.00</td>
<td>E15</td>
<td>0.2</td>
<td>0.1</td>
<td>0.1</td>
<td>0.13</td>
<td>0.05</td>
</tr>
<tr>
<td>E6</td>
<td>0.16</td>
<td>0.15</td>
<td>0.12</td>
<td>0.14</td>
<td>0.01</td>
<td>E16</td>
<td>0.3</td>
<td>0.2</td>
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<td>0.23</td>
<td>0.05</td>
</tr>
<tr>
<td>E7</td>
<td>0.15</td>
<td>0.20</td>
<td>0.18</td>
<td>0.18</td>
<td>0.02</td>
<td>E17</td>
<td>0.2</td>
<td>0.14</td>
<td>0.18</td>
<td>0.17</td>
<td>0.03</td>
</tr>
</tbody>
</table>

TABLE II PROBABILITY VALUE OF THE UNIFORM TRIANGLE FUZZY DATE

<table>
<thead>
<tr>
<th>Code</th>
<th>Probability value</th>
<th>Code</th>
<th>Probability value</th>
<th>Code</th>
<th>Probability value</th>
</tr>
</thead>
<tbody>
<tr>
<td>E1</td>
<td>(0.027, 0.09, 0.027)</td>
<td>E7</td>
<td>(0.06, 0.18, 0.06)</td>
<td>E13</td>
<td>(0.042, 0.064, 0.042)</td>
</tr>
<tr>
<td>E2</td>
<td>(0.021, 0.08, 0.021)</td>
<td>E8</td>
<td>(0.08, 0.11, 0.08)</td>
<td>E14</td>
<td>(0.042, 0.064, 0.042)</td>
</tr>
<tr>
<td>E3</td>
<td>(0.06, 0.12, 0.06)</td>
<td>E9</td>
<td>(0.015, 0.092, 0.015)</td>
<td>E15</td>
<td>(0.15, 0.13, 0.15)</td>
</tr>
<tr>
<td>E4</td>
<td>(0.05, 0.10, 0.20)</td>
<td>E10</td>
<td>(0.07, 0.08, 0.10)</td>
<td>E16</td>
<td>(0.15, 0.23, 0.15)</td>
</tr>
<tr>
<td>E5</td>
<td>(0.015, 0.094, 0.015)</td>
<td>E11</td>
<td>(0.11, 0.20, 0.29)</td>
<td>E17</td>
<td>(0.09, 0.174, 0.09)</td>
</tr>
<tr>
<td>E6</td>
<td>(0.051, 0.14, 0.051)</td>
<td>E12</td>
<td>(0.06, 0.07, 0.12)</td>
<td>E18</td>
<td>(0.18, 0.283, 0.18)</td>
</tr>
</tbody>
</table>
VIII. REFERENCES


IX. BIOGRAPHIES

Tong Wu received a Master degree from Huazhong University of Science & Technology (HUST) in China in 2001. He is presently a doctor candidate at the department of Electrical Power Engineering in HUST. His research interests are automations of power systems, power system protective relaying and control.

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