I. INTRODUCTION

MANY complex and difficult circuit analysis and design tasks are performed with CAD tools, which require accurate component models to achieve accurate circuit simulation results. Currently, the most common nine-element PI model of inductors [1] is narrow-banded. Available bandwidth in the model can be increased by using relatively more complex model configurations such as a modified T configuration [2], [3] or a two-PI configuration [4]. A simpler approach is increasing the number of elements in the PI model [4]–[7]. However, as elements are increased, model parameter extraction becomes more difficult and complex, potentially requiring time-consuming optimization. Modeling methods based on electromagnetic (EM) simulation have a large computational load [5]. Moreover, the resultant frequency-dependent physical models are difficult to integrate into SPICE-compatible simulators.

The elements in the wideband compact model proposed in this study (Fig. 1(a)) are identical to those in the PI model [1] except for an $R_p$ element representing high-frequency loss and an $L_g$-$C_g$-$R_g$ resonator representing higher-order transmission line effects on substrate impedance. The two-step model extraction procedure exploits three $Q$ factors to develop extraction formulations of model elements without optimization. The model can be integrated into SPICE-compatible simulators such that the values of its elements are independent of frequency. The proposed model was validated using CMOS spiral inductors with an octagonal layout, as shown in Fig. 1(b).

II. MODEL EXTRACTION PROCEDURE

A. Step 1 Extraction at Low Frequency Condition

When operating at a frequency much lower than either of the two resonant frequencies of $L_C$ and $L_g$ ($C_{11}$+$C_{12}$), the proposed model can be simplified as a simple PI model, as shown in Fig. 2(a). After the two-port measured $S$ parameters are converted into $Z$ and $Y$ parameters, the four elements in Fig. 2(a) can be determined from the following admittance parameters:

$$C_{21} = \frac{1}{\omega^2 \Im{[Y_{11} + Y_{12}]}}$$  \hspace{0.5cm} \text{at low frequency} \hspace{1cm} (1)

$$C_{22} = \frac{1}{\omega^2 \Im{[Y_{22} + Y_{12}]}}$$  \hspace{0.5cm} \text{at low frequency} \hspace{1cm} (2)

$$Z_c = R_c + j\omega L_c = \frac{1}{Y_{12}}$$  \hspace{0.5cm} \text{at low frequency} \hspace{1cm} (3)

B. Step 2 Extraction at High Frequency Condition

The proposed model uses elements other than the above four elements to capture high frequency effects [6]. To simplify the extraction of these elements, the proposed model is decomposed...
according to the following relations into two one-port equivalent circuits [2], [3]:

\[ Y_A = Z_A^{-1} = (Z_{11} + Z_{22} - 2Z_{12})^{-1} = \frac{1}{R_c + j\omega L_c + j\omega C_p + \frac{1}{R_p}} + \frac{j\omega}{C_x1C_x2} \left( \frac{C_x1C_x2}{C_x1 + C_x2} \right) \]

\[ Z_B = Y_B^{-1} = (Y_{11} + Y_{22} + 2Y_{12})^{-1} = \left( \frac{j\omega C_g + \frac{1}{j\omega L_g} + \frac{1}{R_g}}{1} \right)^{-1} + \frac{1}{j\omega(C_x1 + C_x2)}. \]

Fig. 2(b) and (c) present the two one-port equivalent circuits with an input admittance of \( Y_A \) and an input impedance of \( Z_B \), respectively. By relating Fig. 1(a) to Fig. 2(b) and (c), \( Y_A \) can be considered the input admittance as seen when looking into port 1 with an open circuit in node M and a short circuit from port 2 to the ground, and \( Z_B \) can be considered the input impedance as seen when looking into port 1 with a short circuit from port 2 to port 1. With knowledge of the four elements \( L_c, R_c, C_x1 \) and \( C_x2 \), which were extracted from Step 1, the remaining five elements, \( C_p, R_p, L_g, R_g \), and \( C_g \), can be obtained using \( Y_A \) and \( Z_B \) from Step 2.

Another critical extraction procedure involves three \( Q \) factors of an inductor, which are defined as

\[ Q(\omega) = -\frac{\text{Im}\{Y_{11}(\omega)\}}{\text{Re}\{Y_{11}(\omega)\}} \]

\[ Q_A(\omega) = -\frac{\text{Im}\{Y_A(\omega)\}}{\text{Re}\{Y_A(\omega)\}} \]

\[ Q_B(\omega) = -\frac{\text{Im}\{Y_B(\omega)\}}{\text{Re}\{Y_B(\omega)\}}. \]

Fig. 3 presents the frequency responses of the three \( Q \) factors for a 5 nH CMOS spiral inductor. The self-resonant frequencies \( (f_{ra}, f_{rb}, \text{ and } f_{sr}) \) are determined from the zero positions of the \( Q \) factors to extract the reactive elements. At the angular frequency \( \omega_{ra} = 2\pi f_{ra} \), the imaginary part of (1) is zero, and solving this condition for \( C_p \) yields

\[ C_p = -\frac{1}{\omega_{ra}^2 L_c} \left( \frac{C_x1C_x2}{C_x1 + C_x2} \right). \]

The admittance parameter \( Y_{11} \) of the two-port inductor model neglecting the loss is

\[ Y_{11} = \frac{1}{j\omega L_c} + j\omega \left( \frac{C_p + \frac{C_x1C_x2}{C_x1 + C_x2}}{C_x1 + C_x2} \right) \]

\[ + \frac{\omega C_x3^2 Y_{11}'}{C_x1 + C_x2} \left( \frac{C_p + \frac{C_x1C_x2}{C_x1 + C_x2}}{C_x1 + C_x2} \right)^{-1} \]

\[ \left( \frac{C_x1 + C_x2}{C_x1 + C_x2} \right). \]

where

\[ Y_{11}' = \frac{\omega C_x3^2 Y_{11}'}{C_x1 + C_x2} \left( \frac{C_p + \frac{C_x1C_x2}{C_x1 + C_x2}}{C_x1 + C_x2} \right)^{-1} \]

\[ \left( \frac{C_x1 + C_x2}{C_x1 + C_x2} \right). \]

The imaginary part of (10) vanishes at the angular frequency \( \omega_{sr} = 2\pi f_{sr} \), and so

\[ L_g \]

\[ = \frac{(\omega_{ra}^2 - \omega_{rb}^2)\omega_{sr}^2 L_c(C_x1 + C_p) - 1}{\omega_{ra}^2 - \omega_{rb}^2 + \omega_{sr}^2 L_c(C_x1 + C_p - C_x2 + C_p(C_x1 + C_x2))}. \]

Since \( \omega_{rb} = 2\pi f_{rb} \) is the angular resonant frequency of the \( L_g R_g C_g \) resonator, \( C_g \) can be further determined using

\[ C_g = \frac{1}{\omega_{rb} L_g}. \]

With respect to the resistive elements, the peak \( Q_A \) factor value \( Q_a \) at the angular frequency \( \omega_a = 2\pi f_a \) is used to extract \( R_p \) as follows:

\[ R_p = \frac{Q_a(R_p^2 + \omega_{ra}^2 L_c^2)}{Q_aR_c + \omega_{ra} L_c - \omega_{ra}^2 (R_p^2 + \omega_{ra}^2 L_c^2) \left( \frac{C_p + \frac{C_x1C_x2}{C_x1 + C_x2}}{C_x1 + C_x2} \right)^{-1}}. \]

In Fig. 3, \( Q_B' \) is defined as the quality factor of the \( L_g R_g C_g \) resonator. Similarly, the peak \( Q_b' \) factor value \( Q_b \) at the angular frequency \( \omega_b = 2\pi f_b \) is used to extract \( R_g \) as follows:

\[ R_g = \frac{Q_b \omega_b L_g}{1 - \omega_b^2 C_g L_g}. \]
Fig. 4. Comparison of (a) quality factor and (b) series inductance and resistance, between measurement and the proposed model against the models studied in [1] and [7] for the 5 nH 90 nm CMOS spiral inductor.

Fig. 5. Measured and modeled results for the magnitude of $S_{21}$ (a) and $S_{11}$ (b). (a) $S_{21}$ (b) $S_{11}$.

III. MODEL VERIFICATION AND DISCUSSION

Table I lists the layout parameters and the model parameters extracted using the proposed method for the three spiral inductors realized in a 90 nm single-poly nine-metal (1P9M) CMOS process. The substrate is 300 μm thick silicon with a high resistivity of 0.1 Ωm. Above the substrate is 0.35 μm thick field oxide. The metal layers are copper. The inductor uses the top metal layer with a thickness of 3.3 μm for winding and the second top metal layer with a thickness of 0.8 μm for an underpath. The inter-metal dielectric (IMD) is 10 μm thick with a dielectric constant of 4.2. Notably, the high-resistivity silicon substrate makes the internal inductance associated with the silicon substrate nearly invariant with frequency. Therefore, the extracted inductances in the model can be considered frequency independent over a wide frequency range [8].

To compare the proposed model with other reported models and measurement, Fig. 4(a) and (b) present the modeled Q factor and the extracted inductance and resistance, respectively, of the 5 nH CMOS inductor. Clearly, the nine-element PI model described by Lee [1] is less accurate compared to the 12-element model presented by Chen [7]. The proposed nine-element model achieves comparable or even better performance mainly because the $L_3-C_3-R_3$ resonator functions as the additive term for a third-order approximation of the transmission-line effects on the substrate impedance. Notably, a similar method devised by Lee [6] established a wideband equivalent circuit for the intrinsic inductor impedance. Based on the values extracted for the elements presented in Table I, the modeled and measured $S$-parameters in Fig. 5 confirm the accuracy of the proposed model.

IV. CONCLUSION

The proposed model uses only nine elements, which is the same number used in the conventional PI model, but it achieves relatively higher wideband accuracy. Good agreement between measured and modeled results validates the proposed model.

REFERENCES